PROPER MOTIONS OF STARS IN THE REGION OF THE ORION ASSOCIATION

By

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We present results from a proper motion study of approximately 2000 stars in the Region of the Orion Association Ia and Ib subgroups. Past studies in this region have been restricted for a number of reasons: the large angular size of the association, the lack of computational power and the high observational priority that the forming subgroups Ic and Id have received.

Our observational material spans 100 years, consisting of photographic plates taken in 1955–6 and 1992 at the McCormick Observatory and newly reduced Astrographic Catalogue positions from around 1900. The second and third epoch plates were measured on the University of Minnesota Automated Plate Scanner in two directions with two threshold levels. This provided us with an average of three position estimates per image with an image centering precision of 1–2 microns.

We reduced the measured rectangular coordinates to equatorial coordinates by the overlapping plate technique. This technique takes advantage of the extensive multiple coverage, and uses additional information on magnitude effects obtained from objective grating images.

This study will provide observations that can be used to restrict the theories of stellar association formation, runaway stars, evolutionary ages, pre-stellar gas dynamics and galactic kinematics. Through this work we illustrate the potential power of the overlapping plate technique and its application to wide field astrometry.

CHAPTER 1
INTRODUCTION

This study will make two major contributions to astronomy: act as a working example of the power of using an overlapping plate reduction technique and find the proper motions of stars in the region of the Orion association. These observations can be used to increase our understanding of the dynamics of stellar associations. This chapter will introduce the problem, examine the previous work, and briefly outline the proposed reduction technique and the rationale for using it.

Project Overview

I-Orion, at a distance of 450 pc and with over 50 O and B type stars, is a particularly nearby and rich stellar association. It is centered on B1950 right ascension 5^h30^m and declination $-1°$ with a total area of approximately 100 square degrees. The largest projected linear diameter of the association is about 100 pc, and it has an estimated total mass of about 7.6×10^6 solar masses.

Many authors (cf. Blaauw, 1964) have noted that the study of association kinematics provides insight into many problems in astronomy: stellar evolution, galactic kinematics, pre-stellar gas dynamics, origins of runaway stars and the search for planetary disks (especially in young associations / clusters). Even though the potential for application in astronomy is rich, few concerted studies of the proper motions in I-Orion have been carried out.

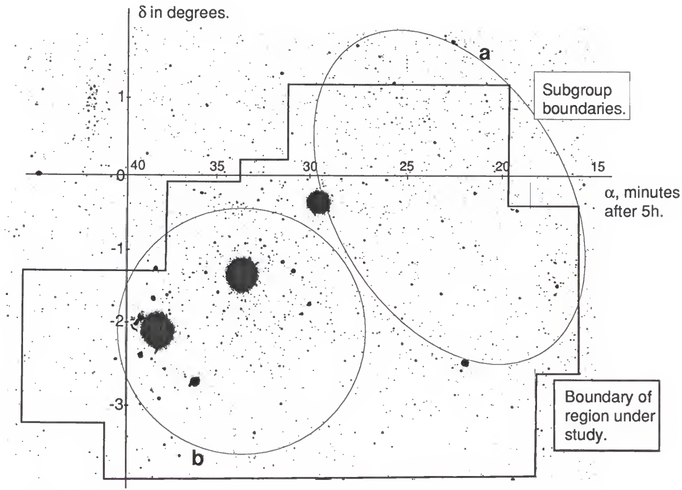
The reason for this lack is probably twofold: First, I-Orion is split into four subgroups, a-d, and within these subgroups the smallest one, Id-Orion, is prioritized

for telescope and research time because this subgroup contains very young stars that have just left their formative stages and are thus important for the study of stellar formation and the search for planetary disks. Second, and probably more to the point, the large angular size of the I-Orion association makes it extremely difficult even to obtain accurate relative positions across the entire field.

Previous investigations of the large subgroups, a and b, in the association were therefore, with two exceptions, restricted to spectroscopic and radial velocity studies. The exceptions looked at proper motions found from different catalogue sources that defined the stellar positions at different epochs. The inherent nonhomogeneity of this data — due to its use of many different sources with systematic and accidental errors — has limited the success in determining proper motion, membership, and kinematic ages. Indeed, as discussed later, the two most recent astronomical studies on these subgroups have called for a more accurate proper motion study.

The present investigation will correct the current lack of accurate proper motion studies. We will determine the proper motions of stars in the region outlined on figure 1 almost complete down to 12th magnitude. To attain consistency across the region, we will use a sophisticated overlapping plate reduction technique that reduces the whole region in order to provide positions for all the stars simultaneously, consistent in a least squares sense, at one epoch.

This is only possible now because of the recent rapid growth in fast, large memory computers. The region under study covers part of both subgroup a and b, extends over 30 square degrees and includes more than 2000 stars.



(Starchart taken from the Vehrenberg 1950.0 Atlas Stellarum. Subgroup outlines taken from Blaauw 1988.)

Figure 1: The Subgroups and Region under Study in the I-Orion Association

Previous Research on Associations; Rationale for this Investigation

Ever since the first determination of spectral classifications for bright stars, the existence of loose groups of O and B types stars has been known (Eddington, 1914). In order of increasing distance the groups IC 2602, Scorpio-Centaurus, II-Perseus, I-Orion, I—Lacerta, NGC 7160, NGC 2264, III-Cepheus, I-Cepheus, NGC 1502, NGC 2169 are all within one kiloparsec of the solar neighborhood. Most of these groups stand out conspicuously as bright configurations in the night sky.

Astronomers carried out considerable research to explain the existence of these groups. Kapteyn (1914) studied them when stars were still believed to burn helium. He produced a whole-sky map of their positions. Collinder (1931) originally classified subgroups of these groups as clusters. Dynamical studies by Bok (1934) and Mineur (1939) showed these groups must be unstable. Therefore, by implication the groups must be young.

Ambartsumian is credited with first coining the name 'O associations' for these groups. In 1952 he described them as follows:

> O-associations are stellar systems, where the partial density of B2 stars is larger than the average field density of these stars, in such a way that this difference cannot be explained by chance fluctuations; moreover, O or B0 stars are present. The properties of O-associations may be described as follows:
>
> 1. the linear diameters range between 30 and 200 parsecs;
> 2. the associations contain an open star cluster of type O as nucleus;
> 3. they include, besides O — B2 stars, also stars of types later than B2, sometimes even Wolf-Rayet stars, though it is difficult to ascertain the number of faint stars;
> 4. sometimes multiple systems of Trapezium type and star chains may be part of the nuclei;
> 5. hot giants occur also outside the nuclei;
> 6. there are reasons for presuming the O-associations to be unstable systems.

Ambartsumian made extensive investigations of the properties of these associations (cf. Ambartsumian, 1955). He pointed out that the combination of the work of Bok, which showed that associations are not bound by their gravitation, and that associations are roughly spherical in shape, implies that associations must be in a state of expansion. He estimated a typical association age to be $\sim 10^7$ years. This kinematic age agrees well with the evolutionary age of the constituent O and B stars. This work established the existence of O associations as expanding young groups of stars. In 1952 Markarian catalogued O associations and created the International Astronomical Union-recommended nomenclature we use today.

The first direct observational evidence supporting Ambartsumian's hypothesis was by Blaauw (1952), and it was later confirmed by Delhaye and Blaauw (1953). In these studies, the authors used proper motions from meridian–circle catalogues and showed that the ζ Persei association (II-Perseus) was expanding. The expansion age found by this study was 1.3 million years, again agreeing well with the members' evolutionary ages. Blaauw (1964) pointed out that the systematic errors in the catalogues used could mimic the effect of expansion, or even contraction. Therefore, the use of catalogues for measuring an association's expansion is severely limited, because of systematic errors and by the general low sky density.

Fredrick (1956) carried out another investigation of II-Perseus using McCormick and Sproul photographic plates. He reduced the plates using a linear single plate reduction with an averaging of the measured rectangular positions from all the plates exposed at one epoch. The total time base for the study was 38 years. Although the reduction was the best that could be performed with the theoretical and computational

tools available at the time, his choice of a simple model and method is prone to many systematic errors. In using a linear plate model, Fredrick ignored radial terms such as magnitude and coma. These terms, if they physically exist but are not represented in the modelling, can produce an expansion of the association even if one does not exist. However, the result was in reasonable agreement with Blauuw's meridian catalogue study and II-Perseus was confirmed to be in a state of expansion.

This cemented the observational evidence for Ambartsumian's hypothesis of young, expanding associations. The 'discovery' of associations and their expansion stimulated research on these groups. Because of their relative youth and apparent dynamical parameters, studies of them have applications to many other areas of astronomy.

Ambartsumian proposed that associations may contain subgroups. Blaauw's 1953 study on II-Perseus supported this. In 1964 Hardie et al. examined the spectroscopic properties of I-Orion, the results of this study also suggested subgroups. Blauuw in his 1964 review paper outlined the various subgroups of local associations and listed some basic properties of them.

The I-Orion association appeared to have 4 subgroups; figure 1 shows the a and b subgroup boundaries. With a few exceptions, most studies of Orion have been limited to the subgroups c and d (for a review of this research cf. van Altena et al. , 1988). We will be looking at the subgroups a and b, which have received very few proper motions studies (a review of the literature shows only two studies: Lesh, 1968, and Gieseking, 1983). The large angular size the subgroups subtend would have made an accurate astrometric reduction at best difficult, and perhaps impossible, before the advent of fast, large memory computers.

There have been some studies that utilized existing catalogues much in the same way Blaauw carried out his original II-Perseus study. One, that specifically addressed the Ia-Orion subgroup, was by Lesh (1968). This study, like Blaauw's, used meridian-circle catalogue data and found an expansion age of $4.5 \times 10^6 \pm 2.3 \times 10^6$ years. Primarily the large size of this error results from the large errors in the proper motions. In addition, since the total number of stars used to derive this age was only 16, the large error also reflects a problem with small sample size. In this study of I-Orion, we will determine uniformly derived proper motions of over 2000 stars. The limiting factor on the precision of the expansion age will be the contamination by non-members.

Another study that incorporated the proper motions of both the a and b subgroups was a membership study of the whole Orion association by Warren and Hessler (1977, 1978). Their study combined all previous photometric, radial velocity, and proper motion measurements and added new photometric data to construct one homogenous set of data. They found expansion ages in good agreement with Lesh's and posed many unresolved questions: What are the defining boundaries of the subgroups? What are the distances of these subgroups? How homogenous is the association? **One of the conclusions in this study was that more complete studies of proper motions and radial velocities are needed for the association, especially in the regions of subgroup a and b.**

Gieseking (1983) carried out an extensive radial velocity study of Ia–Orion. He concluded that it consists of two groups, one of which may be rotating. Further analysis of Hardie's spectroscopic data supported the two-group hypothesis. Like Warren and Hessler, Giesking also concluded that "it would be interesting to investigate whether

these kinematically distinguished (via radial velocities) groups may be recognized also by their proper motions and available space motions. . . . Unfortunately the proper motions available are by no means adequate." This study will provide the proper motions for that investigation.

Advances in Determining Proper Motions and Positions

Proper motions are the time derivatives of the position components of a star in an inertial reference frame or a frame that is a good approximation to one. The standard method for determining proper motions is to find the position of a particular star at two widely spaced epochs in the same frame of reference. The proper motion is closely approximated by the difference of these positions divided by the epoch difference. As we have seen, previous methods used catalogues to define the stellar positions at two epochs. However, this method will only provide proper motions of limited precision.

A more precise method is to photograph the sky at at least two epochs and compare the positions of the stars. This is essentially the method used by Fredrick in his II-Perseus study. However, as we have pointed out, the model his study used to find equatorial coordinates in the sky's frame of reference from the image's measured rectangular coordinate was too simple to account for the possible effects of a star's magnitude or the telescope's coma on the star's rectangular position. (Work carried out by Eichhorn, 1956, revealed the presence of a large coma term in the McCormick telescope.) He made this simplification because the numerical manipulation required to model magnitude and coma effects would have resulted in too large a problem to handle at that time.

In addition, the Orion region in this study covers an area of approximately 10 McCormick photographic plates. The reduction of a number of non-concentric plates covering a large area will lead to varying degrees of precision across the region. Eichhorn (1960, 1984) developed an improved method called the overlapping plate technique. In this method the observer exposes the region such that all plates overlap with at least one other plate (and usually more). This method assures that there will be stars in common to those plates; and because these plates were exposed at essentially the same epoch, the stars will have virtually identical equatorial coordinates.

All the observations of a stellar position are used in the overlap, and they are all simultaneously reduced. Another advantage to the overlap is that all of the stars act as reference material. The equatorial coordinates of a star are tied onto the celestial coordinate system by the use of reference stars. The overlap method uses the multiple images of the non-reference stars to 'lock' plates together; while the reference stars tie the whole plate system to the sky. This reduction technique, if carefully and competently applied, provides very precise, internally consistent, stellar positions in the equatorial frame from photographic plates.

We then derive the stellar proper motions from a comparison of a star's equatorial position at different epochs. In this study we have positions from three epochs, 1900, 1956 and 1992, allowing a weighted least squares determination of the stellar proper motions.

CHAPTER 2
MATHEMATICAL FOUNDATION

To understand the overlapping plate technique it is necessary to outline the theory of least squares. The overlap is essentially a least squares reduction of stellar equatorial coordinates from measured rectangular coordinates of those stars on photographic plates. This chapter lays the mathematical foundation for the development of this technique.

Traditional Least Squares

Consider a set of observations, x_0, which we will assume are unbiased, i.e. without systematic errors. The set of their true values, x, will be the sum of x_0 and a vector ϵ of corrections. Following our assumption of unbiased estimates, and assuming a Gaussian distribution, then we can express the distribution function of the errors as

$$\varphi(\varepsilon) = C \exp\left(-\frac{1}{2}\varepsilon^T \sigma^{-1} \varepsilon\right) \tag{2.1}$$

where, writing all quantities in vector form, then formally

$$x = x_o + \varepsilon, \tag{2.2}$$

and σ is the covariance matrix of x_0 (or obviously ϵ).

Assume further that the measurable quantities, x and certain parameters a, are related by p vector relationships

$$f_\lambda(a, x) = 0, \ (\lambda = 1, ..., p) \tag{2.3}$$

in vector form: $F_{p\times 1}(a, x) = 0$. The equations $f_\lambda(a, x) = 0$ are the condition equations (sometimes called "observation equations") of the problem.

One can never find the true values of the observed quantities, x, but using the principle of maximum likelihood, we can find an estimate of x that is better than the original x_o. In this theory we seek to maximize the value of the likelihood function. This occurs when $\varepsilon^T\sigma^{-1}\varepsilon =$ minimum. The relationship (2.3) between the parameters, a, and the target quantities, x, can thus be rewritten in terms of the errors, ϵ, and observations x_o as

$$F_{p\times 1}(a, x_o + \varepsilon) = 0, \tag{2.4}$$

and these must be **strictly satisfied** at the solution.

Traditional least squares assumes that the observations are of the same precision, uncorrelated and that each equation of condition contains only one component of x_o. Therefore if the variance of any one measurement is σ_{oo}, the covariance matrix will be of the form, σ=σ_{oo} **I**, where **I** is the identity matrix and

$$\varepsilon^T\sigma^{-1}\varepsilon = \frac{1}{\sigma_{oo}}\sum_{\mu=1}^{m}\varepsilon_\mu^2. \tag{2.5}$$

This is the case in traditional least squares, where the values of $\varepsilon^T\sigma^{-1}\varepsilon$ and $\sum\limits_{\mu=1}^{m}\varepsilon_\mu^2$ coincide. After linearization one can write the condition equations in the form

$$\sum_{\nu=1}^{n} a_{\mu\nu}a_\nu + x_{o\mu} = -\varepsilon_\mu \qquad \mu = 1, ..., m \tag{2.6}$$

written in matrix / vector form as $\mathrm{A}a + x_o = -\varepsilon$. The parameters, a, occur linearly in the condition equations, and each equation contains exactly one observation. (In general any number of observations may occur in any of the condition equations.) It is

easy to find the normal equations that give estimates of a

$$
\begin{aligned}
\mathbf{A}a + x_o &= -\varepsilon \\
\rightarrow a^T \mathbf{A}^\mathbf{T} + x_o^T &= -\varepsilon^T \\
\rightarrow a^T \mathbf{A}^\mathbf{T} \mathbf{A} a + 2a^T \mathbf{A}^\mathbf{T} x_o + x_o^T x_o &= \varepsilon^T \varepsilon.
\end{aligned}
\tag{2.7}
$$

(Note: $\epsilon^\mathrm{T}\epsilon$ and $a^\mathrm{T}\mathrm{A}^\mathrm{T}x_0$ are numbers and we may transpose any of their terms without changing their values.) Now differentiate with respect to each component of a and set it equal to zero to find the maximum:

$$
\begin{aligned}
\frac{d(\varepsilon^T\varepsilon)}{da} &= \frac{d(a^T \mathbf{A}^\mathbf{T} \mathbf{A} a + 2a^T \mathbf{A}^\mathbf{T} x_o + x_o^T x_o)}{da} \\
&= 2(\mathbf{A}^\mathbf{T}\mathbf{A}a + \mathbf{A}^\mathbf{T} x_o).
\end{aligned}
\tag{2.8}
$$

If the right hand side of (2.8) is set equal to zero, then the condition becomes $\mathbf{A}^\mathbf{T}\mathbf{A}a = -\mathbf{A}^\mathbf{T}x_o$. By rearranging, we get the traditional least squares solution in its simplest form

$$
a = -(\mathbf{A}^\mathbf{T}\mathbf{A})^{-1}\mathbf{A}^\mathbf{T} x_o. \tag{2.9}
$$

Generalization of the Traditional Method

We can use a more general least squares treatment by dropping some restrictions. Following the procedure of Brown (1955) (see also Jefferys, 1980, 1981; Eichhorn, 1993), we will make use of Laplace multipliers, Λ. At the solution the condition equations, $\mathbf{F}$, equal zero. Therefore the value of $\mathrm{S} = \varepsilon^T\sigma^{-1}\varepsilon$ is the same as the value of

$$
\mathrm{S}^* = \varepsilon^T \sigma^{-1}\varepsilon - 2\Lambda^\mathbf{T} F(a, x_o + \varepsilon). \tag{2.10}
$$

Therefore the values of S and S^* reach their minima at the same values of ϵ and a.

The parameter vector, a, has n components and ϵ has m components, so these two give (m+n) free components. The vector of condition equations, $F=0$, has p components. Since F is restricted to = 0, there are only (m+n-p) free components. Differentiate $\mathbf{S}^*$ w.r.t. the components of ϵ and a, thus

$$\left(\frac{d\mathbf{S}^*}{d\varepsilon}\right) = 2\sigma^{-1}\varepsilon - 2\mathbf{X}^{\mathbf{T}}\mathbf{\Lambda}$$

$$\left(\frac{d\mathbf{S}^*}{da}\right) = \qquad -2\mathbf{A}^{\mathbf{T}}\mathbf{\Lambda} \tag{2.11}$$

where we have used the abbreviations

$$\mathbf{X}_{p\times m} = \left(\frac{dF}{d\varepsilon}\right)_{x=x_o,a=a_o} = \begin{pmatrix} \frac{df_1}{d\epsilon_1} & \cdots & \frac{df_1}{d\epsilon_m} \\ \vdots & \ddots & \vdots \\ \frac{df_p}{d\epsilon_1} & \cdots & \frac{df_p}{d\epsilon_m} \end{pmatrix}_{x=x_o,a=a_o} \tag{2.12}$$

and

$$\mathbf{A}_{p\times n} = \left(\frac{dF}{da}\right)_{x=x_o,a=a_o} = \begin{pmatrix} \frac{df_1}{da_1} & \cdots & \frac{df_1}{da_n} \\ \vdots & \ddots & \vdots \\ \frac{df_p}{da_1} & \cdots & \frac{df_p}{da_n} \end{pmatrix}_{x=x_o,a=a_o} \tag{2.13}$$

At the solution, the following equations must thus be rigorously satisfied;

$$\begin{aligned} \sigma^{-1}\varepsilon &= \mathbf{X}^{\mathbf{T}}\mathbf{\Lambda} && m \quad equations \\ \mathbf{A}^{\mathbf{T}}\Lambda &= 0 && n \quad equations \\ F &= 0 && p \quad equations. \end{aligned} \tag{2.14}$$

The vector Λ has p unknowns; from (2.14) there are exactly as many unknowns as there are equations.

These equations are rigorous at the solution so **A** and **X** must be evaluated at the solution of a and x. These equations are nonlinear and solved by an iterative

procedure (Jefferys, 1981). Assuming some approximate value, a_o, for a and using as approximations to x the observations, x_o, we write $a=a_o+\alpha$ and expand F as a Taylor series.

Formally,

$$a = a_o + \alpha, \qquad x = x_o + \varepsilon$$

$$F(x,a) = F(x_0,a_0) + \mathbf{X_o}\epsilon + \mathbf{A_o}\alpha + O[2] \tag{2.15}$$

where $\mathbf{X_o}$ and $\mathbf{A_o}$ from equations (2.12) and (2.13) and evaluated at x_o and a_o. From the first of equations (2.10) we get $\varepsilon = \sigma\mathbf{X}^{\mathbf{T}}\Lambda$; insert the expansion of F(x,a)=0:

$$\begin{aligned} F = 0 &= F_o + \mathbf{X_o}\varepsilon + \mathbf{A_o}\alpha \\ &\rightarrow F_o + \mathbf{X_o}\sigma\mathbf{X}^{\mathbf{T}}\Lambda + \mathbf{A_o}\alpha = 0 \end{aligned} \tag{2.16}$$

where F_o=F(x_o,a_o) combine with the second of equations (2.10); $\mathbf{A}^{\mathbf{T}}\Lambda = 0$ so that we may write

$$\begin{pmatrix} \mathbf{X}\sigma\mathbf{X}^{\mathbf{T}} & \mathbf{A} \\ \mathbf{A}^{\mathbf{T}} & 0 \end{pmatrix} \begin{pmatrix} \Lambda \\ \alpha \end{pmatrix} + \begin{pmatrix} F_o \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{2.17}$$

These are the generalized normal equations. Note that we need Λ to find ϵ, but if $\mathbf{X}^{\mathbf{T}}\sigma\mathbf{X}$ is nonsingular then we can eliminate Λ and find the corrections to the parameters, α. This may be done as follows:

$$\begin{aligned} &F_o + \mathbf{X}\sigma\mathbf{X}^{\mathbf{T}}\Lambda + \mathbf{A}\alpha = 0 \\ \rightarrow &(\mathbf{X}\sigma\mathbf{X}^{\mathbf{T}})^{-1}F_o + \Lambda + (\mathbf{X}\sigma\mathbf{X}^{\mathbf{T}})^{-1}\mathbf{A}\alpha = 0. \end{aligned} \tag{2.18}$$

Hence

$$\begin{aligned} &\Lambda = -(\mathbf{X}\sigma\mathbf{X}^{\mathbf{T}})^{-1}\mathbf{A}\alpha - (\mathbf{X}\sigma\mathbf{X}^{\mathbf{T}})^{-1}F_o \\ &\rightarrow \Lambda = -\mathbf{W}(\mathbf{A}\alpha + \mathbf{F_o}) \end{aligned} \tag{2.19}$$

where $\mathbf{W} = \left(\mathbf{X}\sigma\mathbf{X}^{\mathbf{T}}\right)^{-1}$, but since $\mathbf{A}^{\mathbf{T}}\Lambda = 0$ we get

$$\mathbf{A}^{\mathbf{T}}(\mathbf{X}\sigma\mathbf{X}^{\mathbf{T}})^{-1}\mathbf{A}\alpha = -\mathbf{A}^{\mathbf{T}}(\mathbf{X}\sigma\mathbf{X}^{\mathbf{T}})^{-1}F_o$$

$$\rightarrow \alpha = -\left[\mathbf{A}^{\mathbf{T}}(\mathbf{X}\sigma\mathbf{X}^{\mathbf{T}})^{-1}\mathbf{A}\right]^{-1}\mathbf{A}^{\mathbf{T}}(\mathbf{X}\sigma\mathbf{X}^{\mathbf{T}})^{-1}\mathbf{F_o}. \tag{2.20}$$

This is a more general solution than that of equation (2.9).

The formal errors ϵ of the observations are then given by

$$\varepsilon = \sigma\mathbf{X}^{\mathbf{T}}\Lambda = -\sigma\mathbf{X}^{\mathbf{T}}\mathbf{W}(\mathbf{A}\alpha + \mathbf{F_o}). \tag{2.21}$$

It can be shown (cf. Brown, 1955; Jefferys, 1980, 1981) that $\left(\mathbf{A}^{\mathbf{T}}\mathbf{W}\mathbf{A}\right)$ is the covariance matrix of α, so that the square roots of its diagonal terms are the standard deviations of the corresponding components of α.

It is instructive to compare the results of the traditional approach with that of the more general approach. In the simplified case, where we use noncorrelated observations of equal precision and when exactly one observation occurs in each equation, **X=-I.** The corrections to the parameters simplify to $\alpha = -\left[\mathbf{A}^{\mathbf{T}}\mathbf{A}\right]^{-1}\mathbf{A}^{\mathbf{T}}F_o$. The observation errors also simplify to $\varepsilon = \left\{\mathbf{I} - \mathbf{A}\left[\mathbf{A}^{\mathbf{T}}\mathbf{A}\right]^{-1}\mathbf{A}^{\mathbf{T}}\right\}F_o$, both of which are traditional least squares solutions, independent of the variances. Also from comparison we can see that the matrix $\left(\mathbf{X}\sigma\mathbf{X}^{\mathbf{T}}\right)^{-1}$ has the same effect as a weight matrix in the traditional approach.

This matrix, $\left(\mathbf{X}\sigma\mathbf{X}^{\mathbf{T}}\right)^{-1}$, is not always nonsingular, as for example if there are equations in F that have no observations but act as parameter constraints. This will not occur in our problem.

In the general case the usual scheme is to set up iterations as follows:

$$\begin{pmatrix} \mathbf{X}_\nu \sigma \mathbf{X}_\nu^{\mathbf{T}} & \mathbf{A}_\nu \\ \mathbf{A}_\nu^{\mathbf{T}} & 0 \end{pmatrix} \begin{pmatrix} \Lambda_{\nu+1} \\ \\ \alpha_{\nu+1} \end{pmatrix} + \begin{pmatrix} F_\nu \\ \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \\ 0 \end{pmatrix} \tag{2.22}$$
$$= \sigma \mathbf{X}_\nu \Lambda_{\nu+1} \varepsilon_{\nu+1}$$

where $\mathbf{X}_\nu = \left(\frac{dF}{dx}\right)_{\substack{a = a_o + \alpha_\nu \\ x = x_o + \varepsilon}}$ $\mathbf{A} = \left(\frac{dF}{da}\right)_{\substack{a = a_o + \alpha_\nu \\ x = x_o + \varepsilon}}$.

This does not always converge, and more sophisticated schemes are usually needed to ensure convergence. For this purpose the important point is that in this scheme the iterations are performed with both the ϵ and the α as variables. We will apply this theory to derive the parameters that define the position of a stellar image on a single photographic plate.

CHAPTER 3
FINDING STELLAR COORDINATES

Now apply the theory of least squares to the determination of equatorial coordinates from the rectangular coordinates of a stellar image on a photographic plate. The first section presents the reduction for single plates, following Eichhorn (1984).

Positions from Single Plates

Consider the data from a single photographic plate. On it, there will be m reference stars and n stars. The position estimates for the reference stars ($\alpha_{\nu c}$,$\delta_{\nu c}$) are available from some catalogue, with their respective variances, $\rho_\nu \cos^2\delta_{\nu c}$ and σ_ν. First make the (reasonably correct) assumption that the $\alpha_{\nu c}$ and $\delta_{\nu c}$ are uncorrelated. The Astrographic Catalogue (or direct measurements from a photographic plate) provides measured rectangular coordinate estimates x_ν and y_ν. The variances υ and ϕ for these coordinate estimates must be obtained somehow.

Let us introduce "standard coordinates," ξ and η, to facilitate the reduction. These are related to the right ascension and declination. The relationship between ξ and η on one hand, and α and δ on the other hand describes the geometry of a gnomonic projection:

$$\begin{pmatrix} \cos\delta\cos(\alpha-\alpha_o) \\ \cos\delta\sin(\alpha-\alpha_o) \\ \sin\delta \end{pmatrix} = \frac{1}{\sqrt{\xi^2+\eta^2+1}} \begin{pmatrix} \cos\delta_o - \eta\sin\delta_o \\ \xi \\ \sin\delta_o + \eta\cos\delta_o \end{pmatrix} \tag{3.1}$$

or inverted

$$\xi = \frac{\cos\delta \sin(\alpha - \alpha_0)}{\sin\delta \sin\delta_0 + \cos\delta \cos\delta_0 \cos(\alpha - \alpha_0)}$$

$$\eta = \frac{\sin\delta \cos\delta_0 - \cos\delta \sin\delta_0 \cos(\alpha - \alpha_0)}{\sin\delta \sin\delta_0 + \cos\delta \cos\delta_0 \cos(\alpha - \alpha_0)}. \tag{3.2}$$

Consider the ideal situation where the optics of the telescope provides exactly the same imaging as a pinhole camera. The rectangular coordinates x_ν,y_ν would thus be related to these standard coordinates ξ,η purely by the focal length and the orientation of the plate in the focal plane. However, a telescope never provides an ideal gnomonic (i.e., pinhole like) projection. The deviations of the telescope's properties from producing an ideal gnomonic projection — these are called aberrations — produce a variety of optical effects that affect the position of a star image. Even if the telescope did produce images exactly in the manner of a pinhole camera, we would still rely on experimental estimates of the telescope's parameters. For example we need the focal length, the location of the plate center and the orientation of the plate with respect to the optical axis. All the deviations and telescope parameters are included in the model and estimated from the observations.

In practice, we assume the relationship between the standard coordinates and the rectangular coordinates of the ν–th star to be of the form

$$\begin{pmatrix} x_\nu \\ y_\nu \end{pmatrix} = s \begin{pmatrix} \xi_\nu \\ \eta_\nu \end{pmatrix} + \Xi_\nu a \tag{3.3}$$

where s is an accurate approximation of the focal length of the telescope, Ξ_ν is the model matrix (with two rows), and a is a vector of parameters (the parameter vector).

The values of the parameter vector a can be found from a linearized least squares analysis if $|\Xi_\nu a| << |s\begin{pmatrix}\xi_\nu \\ \\ \eta_\nu\end{pmatrix}|$.

The model matrix, Ξ, and the concomitant parameter vector, a, can take many forms depending on the properties of the telescope. For this example we will use the simple "six-constant model." In this model relationship (3.3) takes the form

$$\begin{pmatrix} x \\ y \end{pmatrix} = s\begin{pmatrix} \xi \\ \eta \end{pmatrix} + \begin{pmatrix} \xi & \eta & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi & \eta & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ a' \\ b' \\ c' \end{pmatrix}, \tag{3.4}$$

where each of the parameters a ... c has a physical significance.

Parameters c and c' account for differences between the origins of the measurable rectangular image coordinates (x,y) and the standard coordinates (ξ,η). a and b' correct the focal length. Rotation of the plate is corrected for by the parameters b and a'. This relationship (3.4) models any effect which produces a linear relationship between (x,y) and (ξ,η) systems.

For this analysis we use a matrix that contains up to 16 independent parameters, but the underlying reduction remains in principle the same. Obviously, larger and more sophisticated models may account for, and correct more, complicated errors.

Use of the relationship (3.3) will render the condition equations for the reference stars to be of the form

$$H = \begin{pmatrix} \begin{pmatrix} x_1 \\ \\ y_1 \end{pmatrix} - s \begin{pmatrix} \xi_1(\alpha_1, \delta_1) \\ \\ \eta_1(\alpha_1, \delta_1) \end{pmatrix} - \Xi_1 a \\ \begin{pmatrix} x_2 \\ \\ y_2 \end{pmatrix} - s \begin{pmatrix} \xi_2(\alpha_2, \delta_2) \\ \\ \eta_2(\alpha_2, \delta_2) \end{pmatrix} - \Xi_2 a \\ \vdots \qquad \vdots \qquad \vdots \\ \begin{pmatrix} x_m \\ \\ y_m \end{pmatrix} - s \begin{pmatrix} \xi_m(\alpha_m, \delta_m) \\ \\ \eta_m(\alpha_m, \delta_m) \end{pmatrix} - \Xi_m a \end{pmatrix} = 0. \tag{3.5}$$

Note that the Ξ_ν are matrices of dimension $(2\times m)$, where m is the number of components of vector a.

In equation (3.5), the $x_1, y_1, ..., x_m, y_m$ are the observations, and the $\alpha_1, \delta_1 ..., \alpha_m, \delta_m$ and the components of the vector a are the unknowns. For linearizing these equations, we need approximate values for the $\alpha_1, ..., \delta_m$. An analysis of a single plate only incorporates reference stars (i.e., stars for which estimated α and δ are available). The catalogue position estimates of these reference stars provide the additional condition equations which are necessary to make the system determinate:

$$G = \begin{pmatrix} (\alpha_{1c} - \alpha_1)\cos\delta_1 \\ \\ \delta_{1c} - \delta_1 \\ \\ (\alpha_{2c} - \alpha_2)\cos\delta_2 \\ \\ \delta_{2c} - \delta_2 \\ \vdots \qquad \vdots \qquad \vdots \\ (\alpha_{mc} - \alpha_m)\cos\delta_m \\ \\ \delta_{mc} - \delta_m \end{pmatrix} = 0. \tag{3.6}$$

Here, $\alpha_{\nu c}$,...,$\delta_{\nu c}$ are observations (coordinate estimates from a reference catalogue), while α_ν,...,δ_ν are the same unknowns as in H=0 from (3.5).

We combine all the condition equations into one set:

$$F = \begin{pmatrix} H \\ G \end{pmatrix} \tag{3.7}$$

F has the same function as in the previous analysis of the least squares solution. From equation (2.20) the corrections α to the parameters will be given by

$$\alpha = -\left[\mathbf{A}^{\mathbf{T}}(\mathbf{X}\sigma\mathbf{X}^{\mathbf{T}})^{-1}\mathbf{A}\right]^{-1}\mathbf{A}^{\mathbf{T}}(\mathbf{X}\sigma\mathbf{X}^{\mathbf{T}})^{-1}F_o \tag{3.8}$$

where the ν–th element of each matrix is $\mathbf{X} = \left(\frac{dF}{dx}\right)_{a=a_o+\alpha_\nu, x=x_o+\epsilon}$, $\mathbf{A} = \left(\frac{dF}{da}\right)_{a=a_o+\alpha_\nu, x=x_o+\epsilon}$, $F_o = F(x_o, a_o)$.

All the equations contain exactly one observation; therefore, we have the (classical) happy circumstance that $\mathbf{X}_\nu = \left(\frac{dF}{dx}\right) = \mathbf{I}$, the identity matrix. Our first assumption that the observations and catalogue coordinates are uncorrelated gives us a covariance matrix of the form

$$\sigma = \begin{pmatrix} v_1 & & & & & & & & & \\ & \varphi_1 & & & & & & & 0 & \\ & & \ddots & & & & & & & \\ & & & v_m & & & & & & \\ & & & & \varphi_m & & & & & \\ & & & & & \rho_1 & & & & \\ & & & & & & \sigma_1 & & & \\ & & & & & & & \ddots & & \\ & 0 & & & & & & & \rho_m & \\ & & & & & & & & & \sigma_m \end{pmatrix} \tag{3.9}$$

let $\sigma = \begin{pmatrix} \sigma_{xx} & 0 \\ 0 & \sigma_{\alpha\alpha} \end{pmatrix}$.

Equation (3.8) then simplifies to

$$\alpha = -\left[\mathbf{A}^{\mathbf{T}}\sigma^{-1}\mathbf{A}\right]^{-1}\mathbf{A}^{\mathbf{T}}\sigma^{-1}F_o. \tag{3.10}$$

For an initial approximation we take the components of the parameter vector, *a*, to be zero. Now insert in the left hand side of the condition equations (3.5) the observed pairs *x*,*y*, put *a*=0 and calculate the ξ and η from the catalogued α and δ of the reference stars. The right side will not be equal to zero but a vector *d* with, we hope, relatively small components: the residual vector. Also let the corrections to the positions be a vector β defined by small position increments ($\cos\delta d\alpha$, $d\delta$). These are one set of unknowns.

Equation (3.8) thus formally becomes $F = \begin{pmatrix} H \\ G \end{pmatrix} = \begin{pmatrix} d \\ 0 \end{pmatrix}$, where *d* is the residual vector.

With $\alpha_{\mathrm{approx}}=\alpha_{\mathrm{catalogue}}$ and $\delta_{\mathrm{approx}}=\delta_{\mathrm{catalogue}}$ the residual vector for *G* will be zero by definition. The corrections, α, to the parameters become $\alpha = \begin{pmatrix} \beta \\ \\ a \end{pmatrix}$, where β is the vector of corrections to the reference star positions, i.e.

$$\begin{pmatrix} \cos\delta_1 d\alpha_1 \\ d\delta_1 \\ \vdots \\ \cos\delta_m d\alpha_m \\ d\delta_m \end{pmatrix} \tag{3.11}$$

and *a* is the vector of the plate parameters.

Equation (3.8) now simplifies to

$$\begin{aligned} \begin{pmatrix} \beta \\ \\ a \end{pmatrix} &= -[\mathbf{A}^{\mathbf{T}}\sigma^{-1}\mathbf{A}]^{-1}\mathbf{A}^{\mathbf{T}}\sigma^{-1}\begin{pmatrix} d \\ \\ 0 \end{pmatrix} \\ \rightarrow [\mathbf{A}^{\mathbf{T}}\sigma^{-1}\mathbf{A}]\begin{pmatrix} \beta \\ \\ a \end{pmatrix} &= -\mathbf{A}^{\mathbf{T}}\sigma^{-1}\begin{pmatrix} d \\ \\ 0 \end{pmatrix}. \end{aligned} \tag{3.12}$$

First we must investigate the structure of $\mathbf{A}$.

We have defined $\mathbf{A}$ as

$$\mathbf{A} = \left(\frac{dF}{d\alpha}\right) = \left(\frac{d(H,G)}{d(\beta,a)}\right) \tag{3.13}$$

$$= \begin{pmatrix} \frac{dH}{d\beta} & \frac{dH}{da} \\ \frac{dG}{d\beta} & \frac{dG}{da} \end{pmatrix} = -\begin{pmatrix} s\mathbf{B} & \Xi \\ \mathbf{I} & 0 \end{pmatrix}$$

where $\mathbf{B} = \left(\frac{d(\xi_\nu,\eta_\nu)}{d(\alpha_\nu,\delta_\nu)}\right)\begin{pmatrix} \frac{1}{\cos\delta} & 0 \\ 0 & 1 \end{pmatrix}$ and $\Xi = \begin{pmatrix} \Xi_1 \\ \vdots \\ \Xi_{\mathrm{m}} \end{pmatrix}$.

Consider the terms in equation (3.12)

$$\begin{aligned} \mathbf{A}^{\mathrm{T}}\sigma^{-1} &= -\begin{pmatrix} s\mathbf{B} & \Xi \\ \mathbf{I} & 0 \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} \sigma_{xx} & 0 \\ 0 & \sigma_{\alpha\alpha} \end{pmatrix}^{-1} \\ &= -\begin{pmatrix} s\mathbf{B}^{\mathrm{T}} & \mathbf{I} \\ \Xi^{\mathrm{T}} & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{\sigma_{xx}} & 0 \\ 0 & \frac{1}{\sigma_{\alpha\alpha}} \end{pmatrix} \\ &= -\begin{pmatrix} s\mathbf{B}^{\mathrm{T}}\sigma_{xx}^{-1} & \sigma_{\alpha\alpha}^{-1} \\ \Xi^{\mathrm{T}}\sigma_{xx}^{-1} & 0 \end{pmatrix} \end{aligned} \tag{3.14}$$

Multiply this by the matrix $\mathbf{A}$:

$$\mathbf{A}^{\mathrm{T}}\sigma^{-1}\mathbf{A} = \begin{pmatrix} s^2\mathbf{B}^{\mathrm{T}}\sigma_{xx}^{-1}\mathbf{B} + \sigma_{\alpha\alpha} & s\mathbf{B}^{\mathrm{T}}\sigma_{xx}^{-1}\Xi \\ s\Xi^{\mathrm{T}}\sigma_{xx}^{-1}\mathbf{B} & \Xi^{\mathrm{T}}\sigma_{xx}^{-1}\Xi \end{pmatrix}. \tag{3.15}$$

Substituting this into equation (3.8), we get

$$(\mathbf{A}^{\mathrm{T}}\sigma^{-1}\mathbf{A})\begin{pmatrix} \beta \\ \\ a \end{pmatrix} = -\mathbf{A}^{\mathrm{T}}\sigma^{-1}\begin{pmatrix} d \\ \\ 0 \end{pmatrix} \tag{3.16}$$

$$\begin{pmatrix} s^2\mathbf{B}^{\mathrm{T}}\sigma_{xx}^{-1}\mathbf{B} + \sigma_{\alpha\alpha} & s\mathbf{B}^{\mathrm{T}}\sigma_{xx}^{-1}\Xi \\ s\Xi^{\mathrm{T}}\sigma_{xx}^{-1}\mathbf{B} & \Xi^{\mathrm{T}}\sigma_{xx}^{-1}\Xi \end{pmatrix}\begin{pmatrix} \beta \\ \\ a \end{pmatrix} = -\begin{pmatrix} s\mathbf{B}^{\mathrm{T}}\sigma_{xx}^{-1} & \sigma_{xx}^{-1} \\ \Xi^{\mathrm{T}}\sigma_{xx}^{-1} & 0 \end{pmatrix}\begin{pmatrix} d \\ \\ 0 \end{pmatrix}$$

expanding

$$(s^2\mathbf{B}^{\mathrm{T}}\sigma_{xx}^{-1}\mathbf{B}+\sigma_{\alpha\alpha})\beta+s\mathbf{B}^{\mathrm{T}}\sigma_{xx}^{-1}\Xi a=s\mathbf{B}^{\mathrm{T}}\sigma_{xx}^{-1}d \tag{3.17}$$

$$s\Xi^{\mathrm{T}}\sigma_{xx}^{-1}\mathbf{B}\beta+\Xi^{\mathrm{T}}\sigma_{xx}^{-1}\Xi a=\Xi^{\mathrm{T}}\sigma_{xx}^{-1}d.$$

Solve for β;

$$\beta=\left(s^2\mathbf{B}^{\mathrm{T}}\sigma_{xx}^{-1}\mathbf{B}+\sigma_{\alpha\alpha}\right)^{-1}s\mathbf{B}^{\mathrm{T}}\sigma_{xx}^{-1}(d-\Xi a) \tag{3.18}$$

and a

$$s\Xi^{\mathrm{T}}\sigma_{xx}^{-1}\mathbf{B}\Big(\left(s^2\mathbf{B}^{\mathrm{T}}\sigma_{xx}^{-1}\mathbf{B}+\sigma_{\alpha\alpha}\right)^{-1}s\mathbf{B}^{\mathrm{T}}\sigma_{xx}^{-1}(d-\Xi a)\Big)+\Xi^{\mathrm{T}}\sigma_{xx}^{-1}\Xi a = \Xi^{\mathrm{T}}\sigma_{xx}^{-1}d \tag{3.19}$$

whence

$$\Xi^{\mathrm{T}}\Big\{\sigma_{xx}^{-1}-s^2\sigma_{xx}^{-1}\mathbf{B}\left(s^2\mathbf{B}^{\mathrm{T}}\sigma_{xx}^{-1}\mathbf{B}+\sigma_{\alpha\alpha}\right)^{-1}\mathbf{B}^{\mathrm{T}}\sigma_{xx}^{-1}\Big\}\Xi a = \Xi^{\mathrm{T}}\Big\{\sigma_{xx}^{-1}-s^2\sigma_{xx}^{-1}\mathbf{B}\left(s^2\mathbf{B}^{\mathrm{T}}\sigma_{xx}^{-1}\mathbf{B}+\sigma_{\alpha\alpha}\right)^{-1}\mathbf{B}^{\mathrm{T}}\sigma_{xx}^{-1}\Big\}d, \tag{3.20}$$

so that,

$$\Xi^{\mathrm{T}}\mathbf{J}\Xi a=\Xi^{\mathrm{T}}\mathbf{J}d \quad \text{or} \quad a=\left(\Xi^{T}\mathbf{J}\Xi\right)^{-1}\Xi^{\mathrm{T}}\mathbf{J}d \tag{3.21}$$

where $\mathbf{J}=\sigma_{xx}^{-1}-s^2\sigma_{xx}^{-1}\mathbf{B}\left(s^2\mathbf{B}^{\mathrm{T}}\sigma_{xx}^{-1}\mathbf{B}+\sigma_{\alpha\alpha}\right)^{-1}\mathbf{B}^{\mathrm{T}}\sigma_{xx}^{-1}$.

This matrix $\mathbf{J}$ can be simplified using the matrix inversion lemma

$$\mathbf{J}=\left(\sigma_{\mathbf{xx}}+\mathbf{B}\sigma_{\alpha\alpha}^{-1}\mathbf{B}^{\mathrm{T}}\right)^{-1} \tag{3.22}$$

which is a block diagonal consisting of (2×2) blocks on the main diagonal. The parameter vector, a, more explicitly is

$$a=\left(\sum_{\nu=1}^{m}\Xi_{\nu}^{\mathrm{T}}\mathbf{J}_{\nu}\Xi_{\nu}\right)^{-1}\sum_{\nu=1}^{m}\Xi_{\nu}^{\mathrm{T}}\mathbf{J}_{\nu}d_{\nu}. \tag{3.23}$$

Now we must find more explicit expressions for the matrix **J**. We could go directly to equation (3.22) but this would entail evaluating **B**. To evaluate **B** from the initial formula is simple but requires first approximations for the spherical coordinates, α,δ. It is better to express the matrix **B** in terms of quantities we already have estimates for, e.g., ξ and η for which first estimates are just $\frac{1}{s}x$ and $\frac{1}{s}y$. In the case of the astrographic catalogue, estimates of the parameters give very close estimates for the ξ and η in terms of x and y.

Eichhorn (1985) showed if we define the following quantities,

$$\begin{aligned} R &= \sqrt{(1+\xi^2+\eta^2)} \\ T &= \sqrt{\xi^2 + (\cos\delta_o - \eta\sin\delta_o)^2} \\ S &= (\xi R^2 \sin\delta_o)/T \\ U &= (\xi R(\sin\delta_o + \eta\cos\delta_o))/T \\ V &= (R^2(\cos\delta_o - \eta\sin\delta_o))/T \\ W &= ((\xi^2 R\cos\delta_o)/T) + V/R \end{aligned} \tag{3.24}$$

we may write the matrix $\mathbf{B} = \begin{pmatrix} W & -S \\ U & V \end{pmatrix}$. If one further introduces the quantities

$$\begin{aligned} Q &= -UW\rho + SV\sigma \\ Y &= U^2\rho + V^2\sigma \\ Z &= W^2\rho + S^2\sigma \end{aligned} \tag{3.25}$$

where $\rho = \sigma_\alpha$ and $\sigma = \sigma_\delta$. Then from equation (3.22) ,

$$\mathbf{J} = \frac{1}{\nu\varphi + Y\nu + Z\varphi + R^6\sigma\rho} \begin{pmatrix} \varphi + Y & Q \\ Q & \nu + Z \end{pmatrix}. \tag{3.26}$$

Finally we can also use the quantities above to simplify the corrections to the positions, β; again from Eichhorn (1985),

$$\beta = \frac{1}{s(\nu\varphi + R^6\rho\sigma + Y\nu + Z\varphi)} \\ \begin{pmatrix} \rho & 0 \\ 0 & \sigma \end{pmatrix} \left[\begin{pmatrix} W & U \\ -S & V \end{pmatrix} \begin{pmatrix} \varphi & 0 \\ 0 & \upsilon \end{pmatrix} + R^3 \begin{pmatrix} \sigma & 0 \\ 0 & \rho \end{pmatrix} \begin{pmatrix} V & S \\ -U & W \end{pmatrix} \right] (d - \Xi a). \tag{3.27}$$

Example

This completes, in essence, the theoretical analysis of a single plate. It is helpful to summarize what the steps are in extracting the position estimates from an astrographic plate as in the immediate problem.

Combine the published spherical coordinates of the reference stars (α_c,δ_c) with the published plate center (α_o,δ_o) in the trigonometric relations (3.2) and calculate the standard coordinates (ξ',η'). We use the estimated plate parameters from the AC to find approximate rectangular coordinates (x',y') for the equatorial coordinates of the reference stars. Compare the measured coordinates (x,y) of the AC and, using the magnitude as an independent check, identify the reference stars.

After this process we have measured coordinates (x,y) matched with the reference stars' spherical coordinates, (α_c,δ_c). Also for each plate we have initial estimates of the center and the parameters. The next step is to find approximate standard coordinates using

$$\begin{pmatrix} \xi' \\ \eta' \end{pmatrix} = \frac{1}{s}\left\{ \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} a_3 \\ a_6 \end{pmatrix} \right\} \tag{3.28}$$

where the parameters represent any shift of the zero point from the initial estimates. We use these approximate values (ξ',η') for no other purpose than to calculate the matrix

J from equation (3.26) and define the matrix Ξ in equation (3.23). The approximate values (ξ', η') are fully sufficient for these calculations.

Standard coordinate estimates (ξ, η) are found from equation (3.2) using the spherical coordinates of the reference stars with the assumed plate centers. We use these values to calculate the residual vector, d. If we assume an initial value of zero for the parameters, then

$$d = \begin{bmatrix} x_1 - s.\xi_1 \\ y_1 - s.\eta_1 \\ \vdots \\ x_m - s.\xi_m \\ y_m - s.\eta_m \end{bmatrix} \tag{3.29}$$

and from equation (3.23), for the reference stars on the plate,

$$a = \left(\sum_{\nu=1}^{m} \Xi_\nu^T J_\nu \Xi_\nu \right)^{-1} \sum_{\nu=1}^{m} \Xi_\nu^T J_\nu d_\nu \tag{3.30}$$

we find the vector a. As we have assumed initial approximations in the residual vector to be zero, this a will be the actual parameters, except a(3) and a(6) will be corrections to the originally assumed values.

We have found the single plate parameter estimates. Using these we can estimate from the rectangular coordinates (x,y) published in the AC, the standard coordinates of the field stars. These in turn provide spherical coordinates using the inverse of the gnomonic relation. We can now use the overlapping plate technique to improve on these first estimates of the field stars' equatorial coordinates.

Positions from Overlapping Plates

Solution with Unconstrained Parameters

Single plate solutions do not use all the available information. The most important information not used is the constraint that the same star on two plates of the same epoch must have the same set of equatorial coordinates, because a star can only be at one place at one time. In a single plate solution, the data on each plate give independent "best" estimates that can be then averaged to give a "combined best" (even "bester"?) estimate.

This process leads to systematic errors because the available plate parameter estimates are not the true ones (Eichhorn and Wiiliams, 1963). The individual parameter errors generate systematic errors in the star positions calculated with them. This is aggravated by ignoring the above constraint; enforcing more constraints generates more accurate parameters. An overlap solution enforces this constraint, producing more accurate plate parameters, thus inevitably leading to smaller systematic errors in the star positions. The experience of many investigators has shown however that the reduction model (i.e. the matrix Ξ) must be carefully chosen to be the most realistic available.

Assume that there are altogether

1. n at least partially overlapping plates, which are numbered ν=1,...,n,
2. m stars in the region which are either on more than one plate or a reference star; μ is the current star number,
3. m_r of the m stars in the region are reference stars,
4. The numbers of the reference stars are $\mu_{r_1}, \ldots, \mu_{r_{m_r}}$,
5. m_ν stars on the νth plate.

The measures (estimated) rectangular coordinates of the μth star on the νth plate are $x_{\mu\nu}$, $y_{\mu\nu}$.

The equations of conditions are identical with those in a single plate solution. The first step in the overlap is to find estimates of the spherical coordinates α and δ of the field stars from single plate reductions and average those that appear on more than one plate. This yields the required estimated equatorial coordinates for all stars that are in turn used to calculate the standard coordinates. *There must be no loss of significant figures in this process.*

It is essential, at this point, to use the same estimates, $\alpha_{\mu o}$,$\delta_{\mu o}$, for calculating the standard coordinates, $\xi_{\mu\nu}$,$\eta_{\mu\nu}$, of each star. For the reference stars, the catalogued estimates, $\alpha_{c\mu}$,$\delta_{c\mu}$, should be used as initial approximations to these stars' equatorial coordinates.

The condition equations, generated by the measurable coordinates *x,y* for the νth plate, will therefore be of the form

$$H = \begin{pmatrix} \begin{pmatrix} x_{\mu_{\nu_1}\nu} - s\xi_{\mu_{\nu_1}\nu} \\ \\ y_{\mu_{\nu_1}\nu} - s\eta_{\mu_{\nu_1}\nu} \end{pmatrix} - \Xi_{\mu_{\nu_1}\nu} a_\nu \\ \begin{pmatrix} x_{\mu_{\nu_2}\nu} - s\xi_{\mu_{\nu_2}\nu} \\ \\ y_{\mu_{\nu_2}\nu} - s\eta_{\mu_{\nu_2}\nu} \end{pmatrix} - \Xi_{\mu_{\nu_2}\nu} a_\nu \\ \vdots \\ \begin{pmatrix} x_{\mu_{\nu m_\nu}\nu} - s\xi_{\mu_{\nu m_\nu}\nu} \\ \\ y_{\mu_{\nu m_\nu}\nu} - s\eta_{\mu_{\nu m_\nu}\nu} \end{pmatrix} - \Xi_{\mu_{\nu m_\nu}\nu} a_\nu \end{pmatrix} = 0 \tag{3.31}$$

$$\nu = 1, ..., n.$$

Note that equations (3.31) will be satisfied only for the true values of the measurable coordinates x and y and the final estimates for the plate parameters a_ν.

In these equations, we have assumed that the (most likely nonconsecutive) numbers of the stars on the νth plate run from $\mu_{\nu_1}, ..., \mu_{\nu_{m_\nu}}$, which necessitates the complicated indexing. Note that each μ_{ν} is just one natural number; an image of the same star on plates ν and ν' will yield two numbers κ and λ such that $\mu_{\nu_\kappa} = \mu_{\nu_\lambda}$.

Assign the numbers μ to the stars in some organized fashion, for example in order of increasing right ascension. The stars next to each other are then likely to be measured on plates that overlap one another at least partially. This will be elaborated on in the example given below.

Concerning the reference star condition equations, it is best to write the equations corresponding to G=0 for all reference stars together. Otherwise if we write G after the equations (3.31), it will require an awkward criterion to make sure than the equation G_{μ}=0 is set up only once for each reference star.

The measured coordinates of the stars' images on each plate provide the set of equations

$$H = \begin{pmatrix} H_1 \\ H_2 \\ \vdots \\ H_n \end{pmatrix} = 0. \tag{3.32}$$

The reference stars, that is those that have estimates α_c,δ_c of their spherical coordinates listed in some catalogue, generate, in addition to H=0, the following set of condition equations

$$G = \begin{pmatrix} (\alpha_{c\mu_{r1}} - \alpha_{\mu_{r1}}) \cos \delta_{\mu_{r1}} + \varepsilon_{\mu_{r1}} \\ \delta_{c\mu_{r1}} - \delta_{\mu_{r1}} + \epsilon_{\mu_{r1}} \\ (\alpha_{c\mu_{r2}} - \alpha_{\mu_{r2}}) \cos \delta_{\mu_{r2}} + \varepsilon_{\mu_{r2}} \\ \delta_{\mu_{r2}} - \delta_{\mu_{r2}} + \epsilon_{\mu_{r2}} \\ \vdots \\ (\alpha_{c\mu_{rm_r}} - \alpha_{\mu_{rm_r}}) \cos \delta_{\mu_{rm_r}} + \varepsilon_{\mu_{r1rm_r}} \\ \delta_{c\mu_{rm_r}} - \delta_{\mu_{rm_r}} + \epsilon_{\mu_{rm_r}} \end{pmatrix} = 0 \tag{3.33}$$

where c is a catalogue value. These equations reflect the previous statement that the numbers of the reference stars are $\mu_{r_1}, \ldots, \mu_{r_{m_r}}$, $\mu_{\mathrm{r}1}, \ldots, \mu_{\mathrm{r}m\mathrm{r}}$.

There is an essential difference between equations (3.31) and (3.33). Equation (3.31) holds for the "true" values of the observables, i.e. the rectangular coordinates of the stellar images on the plates. The equations are satisfied rigorously after we have found the "definitive" values (estimates) for the plate parameters a_ν.

Equation (3.33) is somewhat different. Instead of observables (the spherical coordinates themselves), we have used their catalogued estimates; the observables are, therefore, also adjustment parameters. To be able to equate (3.33) to zero we need to add corrections ϵ and ξ to the appropriate catalogue estimates. Normally the numbers $\mu_{\mathrm{rk}}, \mu_{\mathrm{rk+1}}$ of two "neighboring" reference stars in equations (3.33) will not be consecutive. In typical situations only a few of the stars on each plate will be reference stars, and they will usually not have consecutively ordinal numbers. Again this will be illustrated by the example.

Equations (3.31) and (3.33) combine to form the condition equations; $F = \begin{pmatrix} H \\ G \end{pmatrix} = 0$. Since only one observation occurs in each equation the matrix $\mathbf{X}$ (being the Jacobian matrix of the condition equations $F = 0$, with respect to the observables, x) will be the identity matrix. This simplifies our least squares solution, such that,

$$\mathbf{X} = \left(\frac{\partial F}{\partial \mathbf{x}}\right)_{x=x, a=a} = \mathbf{I} \tag{3.34}$$

therefore,

$$\mathbf{X}\sigma\mathbf{X}^{\mathrm{T}} = \sigma = \begin{pmatrix} \sigma_{xx} & 0 \\ 0 & \sigma_{\alpha\alpha} \end{pmatrix}. \tag{3.35}$$

From equation (2.20), the correction to the parameters, α, is given by $\alpha = -\left[\mathbf{A}^{\mathrm{T}}(\mathbf{X}\sigma\mathbf{X}^{\mathrm{T}})^{-1}\mathbf{A}\right]^{-1}\mathbf{A}^{\mathrm{T}}(\mathbf{X}\sigma\mathbf{X}^{\mathrm{T}})^{-1}F_o$. When $\mathbf{X} = \mathbf{I}$ this simplifies to, $\alpha = -(\mathbf{A}^{\mathrm{T}}\sigma^{-1}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\sigma^{-1}F_o$.

The next step is to find the matrix $\mathbf{A} = \left(\frac{dF}{d\alpha}\right)$, where α are the adjustment parameters. Let β be the corrections to the stars' spherical coordinates and a the plate parameters, thus $\alpha = \begin{pmatrix} \beta \\ a \end{pmatrix}$. In this example $|\alpha| << 1$ and if the parameter vector is $a = 0$ then, the values we obtain from equation (2.20), will be the actual parameter values rather than the corrections.

Since $\mathbf{A}$ is the Jacobian matrix of the condition equations with respect to the adjustment parameters, we have

$$\mathbf{A} = \left(\frac{\partial F}{\partial \alpha}\right) = \left(\frac{\partial(H,G)}{\partial(\beta,a)}\right) = \begin{pmatrix} \left(\frac{\partial H}{\partial \beta}\right) & \left(\frac{\partial H}{\partial \mathbf{a}}\right) \\ \left(\frac{\partial G}{\partial \beta}\right) & \left(\frac{\partial G}{\partial \mathbf{a}}\right) \end{pmatrix}. \tag{3.36}$$

From equation (3.32) we have

$$\frac{\partial H_\nu}{\partial \alpha} = \begin{pmatrix} \frac{\partial H_1}{\partial \alpha} \\ \frac{\partial H_2}{\partial \alpha} \\ \vdots \\ \frac{\partial H_n}{\partial \alpha} \end{pmatrix} \tag{3.37}$$

and for any H_ν,

$$\frac{\partial H_\nu}{\partial \alpha} = \left(\frac{\partial H_\nu}{\partial \beta}, \frac{\partial H_\nu}{\partial a}\right). \tag{3.38}$$

In this equation β is the vector of star parameters, $(\alpha_\mu \cos\delta_\mu, \delta_\mu)$, and a is the vector of plate parameters, taken one plate after another: $a^T = \left(a_1^T, a_2^T, \ldots, a_2^T\right)$. We consider the condition equations H_ν=0 which arise from the measured rectangular coordinates

of the star numbers, $\mu_{\nu 1},...,\mu_{\nu m \nu}$ on the νth plate. Break down H_ν further and write

$$H_\nu = \begin{pmatrix} H_1 \\ H_2 \\ \vdots \\ H_n \end{pmatrix}, \tag{3.39}$$

where $H_{\mu_\nu \kappa_\nu}$ is a vector of dimension (2 $\times$ 1).

Since there are altogether m stars, the Jacobian matrix $\frac{\partial H_{\mu_\nu \nu}}{\partial \beta}$ is of dimension (2 $\times$ 2m): $\frac{\partial H_{\mu_{\nu_\kappa} \nu}}{\partial \beta} = \left(0 \ldots 0 \mathbf{B}_{\mu_{\nu_\kappa} \nu} 0 \ldots 0\right)$ where the **0** are matrices of dimension (2 $\times$ 2) and $\mathbf{B}_{\mu_{\nu_\kappa} \nu} = \left(\frac{\partial(\xi_{\mu_{\nu_\kappa} \nu}, \eta_{\mu_{\nu_\kappa} \nu})}{\partial(\alpha_{\mu_{\nu_\kappa} \nu} \cos \delta_{\mu_{\nu_\kappa} \nu}, \delta_{\mu_{\nu_\kappa} \nu})}\right)$, are also of dimension (2 $\times$ 2). The reason for the particular structure of $\left(\frac{\partial H_{\mu_\nu \nu}}{\partial \beta}\right)$ is that each pair ξ,η depends on only one pair α,δ. We get therefore from "each plate"

$$\frac{\partial H_\nu}{\partial \beta} = -s \begin{pmatrix} 0 & \ldots & \mathbf{B}_{\mu_{\nu_1} \nu} & 0 & \ldots & 0 & 0 & \ldots & 0 & \ldots & 0 \\ 0 & \ldots & 0 & 0 & \ldots & \mathbf{B}_{\mu_{\nu_2} \nu} & 0 & \ldots & 0 & \ldots & 0 \\ \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & & \vdots \\ 0 & \ldots & 0 & 0 & \ldots & 0 & 0 & \ldots & \mathbf{B}_{\mu_{\nu_{m_\nu}} \nu} & \ldots & 0 \end{pmatrix} = \mathbf{B}_\nu. \tag{3.40}$$

This matrix is of dimension (2$m\nu$ $\times$ 2m). Each line contains exactly one (2 $\times$ 2) matrix that is not a null matrix; and each column **at most** one (2 $\times$ 2) matrix that is not a null matrix, namely when the an image of the star is measured on the νth plate. One could say that $\mathbf{B}_{\mu\nu}$ is then, and only then, not a null matrix when $\mu \in \{\mu_{\nu_\kappa}\}$, i.e., when the number μ of the star is among those whose images were measured on the νth plate.

If we introduce $\mathbf{B} = \begin{pmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \\ \vdots \\ \mathbf{B}_n \end{pmatrix}$ then we may write

$$\frac{\partial H}{\partial \beta} = -s\mathbf{B}. \tag{3.41}$$

The matrix **B** obviously has the dimensions $(2 \sum_{\nu=1}^{n} m_\nu \times 2m.)$

If we write $\Xi_\nu = \begin{pmatrix} \Xi_{\mu_{\nu 1}\nu} \\ \Xi_{\mu_{\nu 2}\nu} \\ \vdots \\ \Xi_{\mu_{\nu n}\nu} \end{pmatrix}$, which has the dimensions $(2m_\nu \times l_\nu)$ where l_ν is the number of components of the vector a_ν, then we see from equation (3.31) that $\frac{\partial H_\nu}{\partial a} = (0 \ldots 0 \;\; -\Xi_\nu \;\; 0 \ldots 0)$ and

$$\frac{\partial H}{\partial a} = \frac{\partial H_\nu}{\partial(a_1, a_2, \cdots, a_n)} = \begin{pmatrix} \frac{\partial H_1}{\partial a_1} & \frac{\partial H_2}{\partial a_1} & \cdots & \frac{\partial H_\nu}{\partial a_1} \\ \frac{\partial H_1}{\partial a_2} & \frac{\partial H_2}{\partial a_2} & \cdots & \frac{\partial H_\nu}{\partial a_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial H_1}{\partial a_\nu} & \frac{\partial H_\nu}{\partial a_\nu} & \cdots & \frac{\partial H_\nu}{\partial a_\nu} \end{pmatrix} \begin{pmatrix} \Xi_1 & 0 & \ldots & 0 \\ 0 & \Xi_2 & \ldots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \ldots & \Xi_n \end{pmatrix} = -\Xi. \tag{3.42}$$

We further see that

$$\frac{\partial G}{\partial \beta} = \frac{\partial \begin{pmatrix} (\alpha_c - \alpha)\cos\delta \\ \\ \delta_c - \delta \end{pmatrix}}{\partial(\alpha\cos\delta, \delta)} = \qquad -\mathbf{K}, \tag{3.43}$$

where $\mathbf{K}$ is a matrix of dimension $(2m_r \times 2m)$. It has exactly one $\mathbf{I}_2$ in each double line and at most one $\mathbf{I}_2$ per double column, namely for those star numbers that correspond to reference stars. We see therefore that the structure of $\mathbf{K}$ is

$$\mathrm{K} = \begin{pmatrix} 0 & \ldots & 0\,\mathbf{I}_2\,0 & \ldots & 0\,0\,0 & \ldots & 0\,0\,0 & \ldots & 0 \\ 0 & \ldots & 0\,0\,0 & \ldots & 0\,\mathbf{I}_2\,0 & \ldots & 0\,0\,0 & \ldots & 0 \\ & & \vdots & & \vdots & & \vdots & & \vdots \\ 0 & \ldots & 0\,0\,0 & \ldots & 0\,0\,0 & \ldots & 0\,\mathbf{I}_2\,0 & \ldots & 0 \end{pmatrix}. \tag{3.44}$$

Since G is independent of the plate parameters we see that $\frac{\partial G}{\partial a} = 0$ and of dimension $(2m_r \times \sum_{\nu=1}^{n} p_\nu)$.

The matrix $\mathbf{A}$ contains the same derivatives in an overlap solution as a single plate reduction, but its structure is more complicated for an overlap reduction. Generally, $\mathbf{A}$ can be written in the form

$$-\mathrm{A} = \begin{pmatrix} s\mathrm{B} & \Xi \\ \mathrm{K} & 0 \end{pmatrix} \tag{3.45}$$

which is analogous to its form for a single plate reduction. The deeper structure of **A** for an overlap reduction deserves close examination, and will be analyzed in the example.

The matrix **B** will be made up of individual blocks of dimension (2 $\times$ 2), $\mathbf{B}_{\mu\nu}$, at each position where the μth star appears on the νth plate. Each plate will generate m_ν separate lines; size $2\sum_{\nu=1}^{n} m_\nu \times 2m$.

Consider the normal equations $(\mathbf{A}^{\mathrm{T}}\sigma^{-1}\mathbf{A})\alpha = -\mathbf{A}^{\mathrm{T}}\sigma^{-1}F_o$ that is to say

$$(\mathbf{A}^{\mathrm{T}}\sigma^{-1}\mathbf{A})\begin{pmatrix}\beta\\ a\end{pmatrix} = -\mathbf{A}^{\mathrm{T}}\sigma^{-1}F_o. \tag{3.46}$$

Let us therefore investigate the structure of the term $(\mathbf{A}^{\mathrm{T}}\sigma^{-1}\mathbf{A})$. Multiplying out

$$\mathbf{A}^{\mathrm{T}}\sigma^{-1}\mathbf{A} = \begin{pmatrix} s^2\sum_{\nu=1}^{\mathbf{n}} \mathbf{B}_\nu^{\mathrm{T}}\sigma_\nu^{-1}\mathbf{B}_\nu + \mathbf{K}^{\mathrm{T}}\sigma_{\alpha\alpha}^{-1}\mathbf{K} & s\mathbf{B}_1^{\mathrm{T}}\sigma_1^{-1}\Xi_1 & \dots & s\mathbf{B}_{\mathbf{n}}^{\mathrm{T}}\sigma_{\mathbf{n}}^{-1}\Xi_{\mathbf{n}} \\ s\Xi_1^{\mathrm{T}}\sigma_1^{-1}\mathbf{B}_1 & \Xi_1^{\mathrm{T}}\sigma_1^{-1}\Xi_1 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ s\Xi_{\mathbf{n}}^{\mathrm{T}}\sigma_{\mathbf{n}}^{-1}\mathbf{B}_{\mathbf{n}} & 0 & \dots & \Xi_{\mathbf{n}}^{\mathrm{T}}\sigma_{\mathbf{n}}^{-1}\Xi_{\mathbf{n}} \end{pmatrix} \tag{3.47}$$

$$\mathbf{A}^{\mathrm{T}}\sigma^{-1}\mathbf{A} = \begin{pmatrix} \mathbf{L} & s\mathbf{B}^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}\Xi \\ s\Xi^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}\mathbf{B} & \Xi^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}\Xi \end{pmatrix}$$

with $\mathbf{L} = s^2\mathbf{B}^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}\mathbf{B} + \mathbf{K}^{\mathrm{T}}\sigma_\alpha^{-1}\mathbf{K}$.

Consider the term $(\mathbf{B}^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}\mathbf{B})$, and the discussion above of the structure of **B**; on each row there is one line pair for each star, consisting of a (2 $\times$ 2) block, otherwise that row will contain only zeros. On each column there will again be a (2 $\times$ 2) block corresponding to a star that appears on the plate corresponding to the row it appears in. (The diagonal covariance matrix σ will not affect the structure of $\mathbf{B}^{\mathrm{T}}\mathbf{B}$.)

The matrix $\mathbf{B}^{\mathrm{T}}\mathbf{B}$ is a multiplication of two matrices of dimension; $(2m \times 2\sum_{\nu=1}^{n} m_\nu)$ and $(2\sum_{\nu=1}^{n} m_\nu \times 2m)$, resulting in a $(2m \times 2m)$ matrix. The μth row and column pair will have an (2 $\times$ 2) block equal to $\sum_{\nu=1}^{n} \mathbf{B}_{\mu\nu}^{\mathrm{T}}\sigma_{\mathbf{x}\mu\nu}^{-1}\mathbf{B}_{\mu\nu}$.

Consider the product $s\mathbf{B}^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}\Xi$; again, the covariance matrix will not affect the structure. The individual elements, $s\mathbf{B}_{\nu}^{\mathrm{T}}\sigma_{\mathbf{x}\nu}^{-1}\Xi_{\nu}$, are sparse matrices of dimensions $(2m \times l_{\nu})$. Nonzero elements of the product $s\mathbf{B}^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}\Xi$, will occur only where the μth star appears on the νth plate. Thus the structure of this matrix will be similar to that of $\mathbf{B}^{\mathrm{T}}$ with the νth column pair replaced by a line of width l_{ν} , i.e. $\mathbf{B}^{\mathrm{T}}\Xi$ is a $(2m \times 2m)$ matrix.

Finally, the product $\Xi^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}\Xi$ is block diagonal with the individual blocks equal to the products of each plate's model matrices and covariance matrices. This ends the consideration of the left hand side in (3.46).

Consider the right hand side of (3.46). This will be a vector with $2m + \sum_{\nu=1}^{n} l_{\nu}$ components

$$\mathbf{A}^{\mathrm{T}}\sigma^{-1}F = \begin{pmatrix} s\sum_{\nu=1}^{n}\mathbf{B}_{\nu}^{\mathrm{T}}\sigma_{\nu}^{-1}d_{\nu} \\ \Xi_{1}^{\mathrm{T}}\sigma_{1}^{-1}d_{1} \\ \vdots \\ \Xi_{\mathrm{n}}^{\mathrm{T}}\sigma_{\mathrm{n}}^{-1}d_{n} \end{pmatrix} = \begin{pmatrix} s\mathbf{B}^{\mathrm{T}} \\ \Xi^{\mathrm{T}} \end{pmatrix}\sigma_{\mathbf{x}}^{-1}d. \tag{3.48}$$

We arrive at the solution by first eliminating the star parameters and solving for the plate parameters only. Consider (3.46): $(\mathbf{A}^{\mathrm{T}}\sigma^{-1}\mathbf{A})\begin{pmatrix}\beta \\ \mathrm{a}\end{pmatrix} = -\mathbf{A}^{\mathrm{T}}\sigma^{-1}F_{o}$. We can substitute $\mathbf{A}^{\mathrm{T}}\sigma^{-1}\mathbf{A}$ and $\mathbf{A}^{\mathrm{T}}\sigma^{-1}F_{o}$ from equations (3.47) and (3.48) to find that

$$\begin{pmatrix} \mathbf{L} & s\mathbf{B}^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}\Xi \\ s\Xi^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}\mathbf{B} & \Xi^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}\Xi \end{pmatrix}\begin{pmatrix}\beta \\ \mathrm{a}\end{pmatrix} = \begin{pmatrix} s\mathbf{B}^{\mathrm{T}} \\ \Xi^{\mathrm{T}} \end{pmatrix}\sigma_{\mathbf{x}}^{-1}\mathrm{d}, \tag{3.49}$$

or multiplying out

$$\begin{aligned} \mathbf{L}\beta + s\mathbf{B}^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}\Xi\mathrm{a} &= s\mathbf{B}^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}d \\ s\Xi^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}\mathbf{B}\beta + \Xi^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}\Xi\mathrm{a} &= \Xi^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}d. \end{aligned} \tag{3.50}$$

The first row gives

$$\beta = s\mathbf{L}^{-1}\mathbf{B}^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}(d - \Xi a)$$

$$\text{and } \beta = s\left(\delta_{\mu,\mu_n}\sigma_{\mu_n}^{-1} + s^2\sum_{\nu=1}^{n}\mathbf{B}_{\mu\nu}^{\mathrm{T}}\sigma_{\mu\nu}^{-1}\mathbf{B}_{\mu\nu}\right)^{-1}\sum_{\nu=1}^{n}\mathbf{B}_{\mu\nu}^{\mathrm{T}}\sigma_{\mu\nu}^{-1}(d_{\mu\nu} - \Xi_{\mu\nu}a_{\nu}). \tag{3.51}$$

The vector β_{μ}= $(\mathrm{d}\alpha_{\mu}\cos\,\delta_{\mu},\ \mathrm{d}\delta_{\mu})^{\mathrm{T}}$ is different from the quantity β_{ν}. The term δ_{μ,μ_n} is a Kronecker symbol which will be equal to zero if the μth star is not a reference star and equal to one if it is a reference star. The components of the matrix β are the corrections to the star coordinates individually and amount to weighted means of all the frames on which a particular star occurred.

Substituting this into the second row of (3.50) gives

$$\begin{gathered} s^2\Xi^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}\mathbf{B}\mathbf{L}^{-1}\mathbf{B}^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}(d - \Xi a) + \Xi^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}\Xi a = \Xi^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}d \\ \rightarrow \Xi^{\mathrm{T}}(\sigma_{\mathbf{x}}^{-1} - s^2\sigma_{\mathbf{x}}^{-1}\mathbf{B}\mathbf{L}^{-1}\mathbf{B}^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1})(d - \Xi a) = 0 \end{gathered} \tag{3.52}$$

simplifying

$$a = \left(\Xi^{\mathrm{T}}\mathbf{J}'\Xi\right)^{-1}\Xi^{\mathrm{T}}\mathbf{J}'d \tag{3.53}$$

where $\mathbf{J}' = \sigma_{\mathbf{x}}^{-1} - s^2\sigma_{\mathbf{x}}^{-1}\mathbf{B}\mathbf{L}^{-1}\mathbf{B}^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}$.

The inversion lemma cannot be used on $\mathbf{J}'$ because the matrix $\mathbf{K}^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}\mathbf{K}$ in $\mathbf{L}$ is singular. The matrix $\mathbf{J}'$ is not block diagonal but assumes the pattern of $\mathbf{B}^{\mathrm{T}}\mathbf{B}$ because $\mathbf{L}$ is block diagonal. The product $\Xi^{\mathrm{T}}\mathbf{J}'\Xi a$ is 'banded-bordered' and there exist efficient routines to invert it.

It is instructive to examine the form of the Laplace multiplier Λ introduced in equation (2.10). From equation (2.9)

$$\Lambda = -\mathbf{W}(\mathbf{A}\alpha + \mathbf{F}_{\mathrm{o}}) \tag{3.54}$$

where $\mathbf{W} = \left(\mathbf{X}\sigma\mathbf{X}^{\mathrm{T}}\right)^{-1}$. In the overlap case

$$\begin{gathered}\mathbf{X} = \mathbf{I} \\ \therefore \mathbf{W} = \left(\mathbf{X}\sigma\mathbf{X}^{\mathrm{T}}\right)^{-1} = \sigma^{-1}.\end{gathered} \tag{3.55}$$

Substituting this into equation (3.54), we get

$$\Lambda = -\sigma^{-1}(\mathbf{A}\alpha + \mathbf{F_o}) = \sigma^{-1}\mathbf{A}\begin{pmatrix}\beta \\ \mathbf{a}\end{pmatrix} + \sigma^{-1}\mathbf{F_o}, \tag{3.56}$$

and substituting for σ, A and $\mathrm{F_o}$

$$\begin{aligned}\Lambda &= -\left\{\begin{pmatrix}\sigma_{xx}^{-1} & 0 \\ 0 & \sigma_{\alpha\alpha}^{-1}\end{pmatrix}\left[-\begin{pmatrix}s\mathbf{B} & \Xi \\ \mathbf{K} & 0\end{pmatrix}\right]\begin{pmatrix}\beta \\ a\end{pmatrix} + \begin{pmatrix}\sigma_{xx}^{-1} & 0 \\ 0 & \sigma_{\alpha\alpha}^{-1}\end{pmatrix}\begin{pmatrix}d \\ 0\end{pmatrix}\right\} \\ \Lambda &= \begin{pmatrix}s\sigma_{xx}^{-1}\mathbf{B} & \sigma_{\mathbf{xx}}^{-1}\Xi \\ \sigma_{\alpha\alpha}^{-1}\mathbf{K} & 0\end{pmatrix}\begin{pmatrix}\beta \\ a\end{pmatrix} - \begin{pmatrix}\sigma_{xx}^{-1}d \\ 0\end{pmatrix} \\ \Lambda &= \begin{pmatrix}\sigma_{xx}^{-1}(s\mathbf{B}\beta + \Xi a - d) \\ \sigma_{\alpha\alpha}^{-1}\mathbf{K}\beta\end{pmatrix}.\end{aligned} \tag{3.57}$$

From equation (3.5) the formal errors in the observations are therefore

$$\varepsilon = \sigma\mathbf{X}^{\mathrm{T}}\Lambda = \begin{pmatrix}s\mathbf{B}\beta + \Xi a - d \\ \mathbf{K}\beta\end{pmatrix}. \tag{3.58}$$

Example

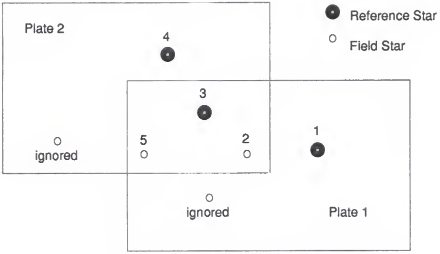


Figure 2: Arrangement of Stars

As in the single plate, it is helpful to summarize what the steps are in evaluating the positions using overlapping plates. Because of the complexity here it is helpful to illustrate the method by a discrete example. In appendix two we give a numerical example of two overlapped plates with ten stars using an unconstrained solution. Here, we show the mathematical constructs resulting from a simple two-plate overlap.

Consider a "six constant" overlap of two plates, both having five stars with three of these in common. There is one reference star among the three common stars; each plate has one other reference star. Altogether, only five stars will enter the calculations, because the two isolated stars on the plates, which are not reference stars, must be ignored. We arrange the stars in order of increasing right ascension, thus m=5 and μ=1...5 and n=2 so that, ν =1,2. Figure 2 shows this arrangement of stars.

On plate 1, the stars are (m_1=4); $1_1=1$, $1_2=2$, $1_3=3$, $1_{m_1}=1_4=5$.

On plate 2, the stars are (m_2=4); $2_1=2$, $2_2=3$, $2_3=4$, $2_{m_2}2_{m2}=2_4=5$

There are thus three reference stars (m_r=3);μ_{r_1}=1, μ_{r_2}=3, $\mu_{r_{m_r}}$=μ_{r_3}=4.

In this case the condition equations will be

$$H = \begin{pmatrix} \\ F \\ \\ G \\ \\ \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} x_{1_1 1} - s\xi_{1_1 1} \\ y_{1_1 1} - s\eta_{1_1 1} \end{pmatrix} - \Xi_{1_1 1} a_1 \\ \begin{pmatrix} x_{1_2 1} - s\xi_{1_2 1} \\ y_{1_2 1} - s\eta_{1_2 1} \end{pmatrix} - \Xi_{1_2 1} a_1 \\ \begin{pmatrix} x_{1_3 1} - s\xi_{1_3 1} \\ y_{1_3 1} - s\eta_{1_3 1} \end{pmatrix} - \Xi_{1_3 1} a_1 \\ \begin{pmatrix} x_{1_4 1} - s\xi_{1_4 1} \\ y_{1_4 1} - s\eta_{1_4 1} \end{pmatrix} - \Xi_{1_4 1} a_1 \\ \begin{pmatrix} x_{2_1 1} - s\xi_{2_1 1} \\ y_{2_1 1} - s\eta_{2_1 1} \end{pmatrix} - \Xi_{2_1 1} a_2 \\ \begin{pmatrix} x_{2_2 1} - s\xi_{2_2 1} \\ y_{2_2 1} - s\eta_{2_2 1} \end{pmatrix} - \Xi_{2_2 1} a_2 \\ \begin{pmatrix} x_{2_3 1} - s\xi_{2_3 1} \\ y_{2_3 1} - s\eta_{2_3 1} \end{pmatrix} - \Xi_{2_3 1} a_2 \\ \begin{pmatrix} x_{2_4 1} - s\xi_{2_4 1} \\ y_{2_4 1} - s\eta_{2_4 1} \end{pmatrix} - \Xi_{2_4 1} a_2 \\ (\alpha_{c1} - \alpha_1)\cos\delta_1 \\ \delta_{c1} - \delta_1 \\ (\alpha_{c3} - \alpha_3)\cos\delta_3 \\ \delta_{c3} - \delta_3 \\ (\alpha_{c4} - \alpha_4)\cos\delta_4 \\ \delta_{c4} - \delta_4 \end{pmatrix}, \text{ First estimate=} \begin{pmatrix} x_{11} - s\xi_{11} \\ y_{11} - s\eta_{11} \\ x_{21} - s\xi_{21} \\ y_{21} - s\eta_{21} \\ x_{31} - s\xi_{31} \\ y_{31} - s\eta_{31} \\ x_{51} - s\xi_{51} \\ y_{51} - s\eta_{51} \\ x_{22} - s\xi_{22} \\ y_{22} - s\eta_{22} \\ x_{32} - s\xi_{32} \\ y_{32} - s\eta_{32} \\ x_{42} - s\xi_{42} \\ y_{42} - s\eta_{42} \\ x_{52} - s\xi_{52} \\ y_{52} - s\eta_{52} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} d_{11} \\ d_{21} \\ d_{31} \\ d_{51} \\ d_{22} \\ d_{32} \\ d_{42} \\ d_{52} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{3.59}$$

Here, the residual is $d = \begin{pmatrix} x - s\xi \\ y - s\eta \end{pmatrix}$; optionally one would use $d = \begin{pmatrix} x - (a\xi + b\eta + c) \\ y - (-b\eta + a\xi + d) \end{pmatrix}$. The first matrices to examine are **B, K,** Ξ and σ . The dimension of the matrix **B** is (16 $\times$ 10)

$$\mathbf{B} = \begin{pmatrix} \mathbf{B}_{11} & 0 & 0 & 0 & 0 \\ 0 & \mathbf{B}_{21} & 0 & 0 & 0 \\ 0 & \mathbf{B}_{22} & 0 & 0 & 0 \\ 0 & 0 & \mathbf{B}_{31} & 0 & 0 \\ 0 & 0 & \mathbf{B}_{32} & 0 & 0 \\ 0 & 0 & 0 & \mathbf{B}_{42} & 0 \\ 0 & 0 & 0 & 0 & \mathbf{B}_{51} \\ 0 & 0 & 0 & 0 & \mathbf{B}_{52} \end{pmatrix}. \tag{3.60}$$

We find the individual elements in the same way as in the single plate solution. Using (3.24), $\mathbf{B}_{\mu\nu} = \frac{d(\xi_{\mu\nu},\eta_{\mu\nu})}{d(\alpha,\delta)}\begin{pmatrix} \frac{1}{\cos\delta} & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} W & -S \\ U & V \end{pmatrix}$ where W,S,U and V are from equation (3.24).

The dimension of the matrix $\mathbf{K}$ is (6×10); $\mathbf{K}$ has the form

$$\mathbf{K} = \begin{pmatrix} \mathbf{I}_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{I}_2 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{I}_2 & 0 \end{pmatrix}. \tag{3.61}$$

The matrix Ξ has dimensions (16×12) and is of the form

$$\Xi^{\mathrm{T}} = \begin{pmatrix} \Xi_{11} & \Xi_{21} & 0 & \Xi_{31} & 0 & 0 & \Xi_{51} & 0 \\ 0 & 0 & \Xi_{22} & 0 & \Xi_{32} & \Xi_{42} & 0 & \Xi_{52} \end{pmatrix} \tag{3.62}$$

where each $\Xi_{\mu\nu}$ is a (2×6) matrix $= \begin{pmatrix} \xi_{\mu\nu} & \eta_{\mu\nu} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi_{\mu\nu} & \eta_{\mu\nu} & 1 \end{pmatrix}$.

The covariance matrix is $\sigma = \begin{pmatrix} \sigma_{xx} & 0 \\ 0 & \sigma_{\alpha\alpha} \end{pmatrix}$ where $\sigma_{\mathbf{x}}$ is a (16×16) matrix and σ_α is a (6×6) matrix:

$$\sigma_{\mathbf{x}} = \begin{pmatrix} \sigma_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{21} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{31} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{32} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{42} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{51} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{52} \end{pmatrix}, \quad \sigma_\alpha = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_3 & 0 \\ 0 & 0 & \sigma_4 \end{pmatrix}. \tag{3.63}$$

In this simple case the matrix $\mathbf{A}$ will look as follows:

$$\mathbf{A} = \begin{pmatrix} \mathbf{B}_{11} & 0 & 0 & 0 & 0 & \Xi_{11} & 0 \\ 0 & \mathbf{B}_{21} & 0 & 0 & 0 & \Xi_{21} & 0 \\ 0 & \mathbf{B}_{22} & 0 & 0 & 0 & 0 & \Xi_{22} \\ 0 & 0 & \mathbf{B}_{31} & 0 & 0 & \Xi_{31} & 0 \\ 0 & 0 & \mathbf{B}_{32} & 0 & 0 & 0 & \Xi_{32} \\ 0 & 0 & 0 & \mathbf{B}_{42} & 0 & 0 & \Xi_{42} \\ 0 & 0 & 0 & 0 & \mathbf{B}_{51} & \Xi_{51} & 0 \\ 0 & 0 & 0 & 0 & \mathbf{B}_{52} & 0 & \Xi_{52} \\ \mathbf{I}_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{I}_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{I}_2 & 0 & 0 & 0 \end{pmatrix}. \tag{3.64}$$

First we find the matrix $\mathbf{L} = s^2\mathbf{B}^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}\mathbf{B} + \mathbf{K}^{\mathrm{T}}\sigma_{\alpha}^{-1}\mathbf{K}$. This is a (10×10) block diagonal matrix which when inverted has the form

$$\mathbf{L}^{-1} = \mathrm{diag}\begin{pmatrix} (s^2\mathbf{B}_{11}^{\mathrm{T}}\sigma_{11}^{-1}\mathbf{B}_{11} + \sigma_1^{-1})^{-1} \\ s^{-2}(\mathbf{B}_{21}^{\mathrm{T}}\sigma_{21}^{-1}\mathbf{B}_{21} + \mathbf{B}_{22}^{\mathrm{T}}\sigma_{22}^{-1}\mathbf{B}_{22})^{-1} \\ (s^2\mathbf{B}_{31}^{\mathrm{T}}\sigma_{31}^{-1}\mathbf{B}_{31} + s^2\mathbf{B}_{32}^{\mathrm{T}}\sigma_{32}^{-1}\mathbf{B}_{32} + \sigma_3)^{-1} \\ (s^2\mathbf{B}_{42}^{\mathrm{T}}\sigma_{42}^{-1}\mathbf{B}_{42} + \sigma_4)^{-1} \\ s^{-2}(\mathbf{B}_{51}^{\mathrm{T}}\sigma_{51}^{-1}\mathbf{B}_{51} + \mathbf{B}_{52}^{\mathrm{T}}\sigma_{52}^{-1}\mathbf{B}_{52})^{-1} \end{pmatrix}^{\mathrm{T}} . \tag{3.65}$$

So that $\mathbf{B}\mathbf{L}^{-1}\mathbf{B}^{\mathrm{T}} =$

$$\begin{pmatrix} \mathbf{B}_{11}\mathbf{L}_{11}^{-1}\mathbf{B}_{11}^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{B}_{21}\mathbf{L}_{22}^{-1}\mathbf{B}_{21}^{\mathrm{T}} & \mathbf{B}_{21}\mathbf{L}_{22}^{-1}\mathbf{B}_{22}^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{B}_{22}\mathbf{L}_{22}^{-1}\mathbf{B}_{21}^{\mathrm{T}} & \mathbf{B}_{22}\mathbf{L}_{22}^{-1}\mathbf{B}_{22}^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{B}_{31}\mathbf{L}_{33}^{-1}\mathbf{B}_{31}^{\mathrm{T}} & \mathbf{B}_{31}\mathbf{L}_{33}^{-1}\mathbf{B}_{32}^{\mathrm{T}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{B}_{32}\mathbf{L}_{33}^{-1}\mathbf{B}_{31}^{\mathrm{T}} & \mathbf{B}_{32}\mathbf{L}_{33}^{-1}\mathbf{B}_{32}^{\mathrm{T}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{B}_{42}\mathbf{L}_{44}^{-1}\mathbf{B}_{42}^{\mathrm{T}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{B}_{51}\mathbf{L}_{55}^{-1}\mathbf{B}_{51}^{\mathrm{T}} & \mathbf{B}_{52}\mathbf{L}_{55}^{-1}\mathbf{B}_{51}^{\mathrm{T}} \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{B}_{51}\mathbf{L}_{55}^{-1}\mathbf{B}_{52}^{\mathrm{T}} & \mathbf{B}_{52}\mathbf{L}_{55}^{-1}\mathbf{B}_{52}^{\mathrm{T}} \end{pmatrix},$$

substituting into equation (3.53) we get $\mathbf{J}' = \sigma_{\mathbf{x}}^{-1} - \mathrm{s}^2\sigma_{\mathbf{x}}^{-1}\mathbf{B}\mathbf{L}^{-1}\mathbf{B}^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}$ or

$$\begin{pmatrix} \mathbf{J}'_{111} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{J}'_{211} & \mathbf{J}'_{212} & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{J}'_{221} & \mathbf{J}'_{222} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{J}'_{311} & \mathbf{J}'_{312} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{J}'_{321} & \mathbf{J}'_{322} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{J}'_{422} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{J}'_{511} & \mathbf{J}'_{512} \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{J}'_{521} & \mathbf{J}'_{522} \end{pmatrix} \tag{3.67}$$

where

$$\mathbf{J}'_{\mathbf{abc}} = \begin{cases} \sigma_{\mathbf{ab}}^{-1} - \mathrm{s}^2\sigma_{\mathbf{ab}}^{-1}\mathbf{B}_{\mathbf{ab}}\mathbf{L}_{\mathbf{aa}}^{-1}\mathbf{B}_{\mathbf{ac}}^{\mathrm{T}}\sigma_{\mathbf{ac}}^{-1} & \text{if b=c} \\ \mathrm{s}^2\sigma_{\mathbf{ab}}^{-1}\mathbf{B}_{\mathbf{ab}}\mathbf{L}_{\mathbf{aa}}^{-1}\mathbf{B}_{\mathbf{ac}}^{\mathrm{T}}\sigma_{\mathbf{ac}}^{-1} & \text{if b=c.} \end{cases} \tag{3.68}$$

This is a 'banded bordered' matrix where the diagonal and adlacent elemtns to the diagonal are also filled on those positions that correspond to stars imaged on more than one plate.

The products needed for the overlap solution are

$$\begin{gathered} \Xi^{\mathrm{T}}\mathbf{J}'\Xi = \Xi^{\mathrm{T}}(\sigma_{\mathbf{x}}^{-1} - \mathrm{s}^2\sigma_{\mathbf{x}}^{-1}\mathbf{B}\mathbf{L}^{-1}\mathbf{B}^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1})\Xi \\ (12 \times 16)(16 \times 16)(16 \times 12) = (12 \times 12) \\ \Xi^{\mathrm{T}}\mathbf{J}'d = \Xi^{\mathrm{T}}(\sigma_{\mathbf{x}}^{-1} - \mathrm{s}^2\sigma_{\mathbf{x}}^{-1}\mathbf{B}\mathbf{L}^{-1}\mathbf{B}^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1})d \\ (12 \times 16)(16 \times 16)(16 \times 1) = (12 \times 1) \end{gathered} \tag{3.69}$$

Substituting from (3.62) and (3.67)

$$\Xi^T J' \Xi = \Xi^T(\sigma_x^{-1} - s^2\sigma_x^{-1} B L^{-1} B^T \sigma_x^{-1})\Xi =$$

$$\begin{pmatrix} \Xi_{11}^T & \Xi_{21}^T & 0 & \Xi_{31}^T & 0 & 0 & \Xi_{51}^T & 0 \\ 0 & 0 & \Xi_{22}^T & 0 & \Xi_{32}^T & \Xi_{42}^T & 0 & \Xi_{52}^T \end{pmatrix}$$

$$\begin{pmatrix} J'_{111} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & J'_{211} & J'_{212} & 0 & 0 & 0 & 0 & 0 \\ 0 & J'_{221} & J'_{222} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & J'_{311} & J'_{312} & 0 & 0 & 0 \\ 0 & 0 & 0 & J'_{321} & J'_{322} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & J'_{422} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & J'_{511} & J'_{512} \\ 0 & 0 & 0 & 0 & 0 & 0 & J'_{521} & J'_{522} \end{pmatrix} \begin{pmatrix} \Xi_{11} & 0 \\ \Xi_{21} & 0 \\ 0 & \Xi_{22} \\ \Xi_{31} & 0 \\ 0 & \Xi_{32} \\ 0 & \Xi_{42} \\ \Xi_{51} & 0 \\ 0 & \Xi_{52} \end{pmatrix} \tag{3.70}$$

$$= \begin{pmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{pmatrix}$$

where

$$\begin{aligned} \Pi_{11} &= \Xi_{11}^T J'_{111}\Xi_{11} + \Xi_{21}^T J'_{211}\Xi_{21} + \Xi_{31}^T J'_{311}\Xi_{31} + \Xi_{51}^T J'_{511}\Xi_{51} = \sum_{\mu\nu} \Xi_{\mu 1}^T J'_{\mu 11}\Xi_{\mu 1} \\ \Pi_{12} &= \Xi_{21}^T J'_{211}\Xi_{22} + \Xi_{31}^T J'_{312}\Xi_{32} + \Xi_{51}^T J'_{512}\Xi_{52} = \sum_{\mu\nu} \Xi_{\mu 1}^T J'_{\mu 12}\Xi_{\mu 2} \\ \Pi_{21} &= \Xi_{22}^T J'_{221}\Xi_{21} + \Xi_{32}^T J'_{321}\Xi_{31} + \Xi_{52}^T J'_{521}\Xi_{51} = \sum_{\mu\nu} \Xi_{\mu 2}^T J'_{\mu 21}\Xi_{\mu 1} \\ \Pi_{22} &= \Xi_{22}^T J'_{222}\Xi_{22} + \Xi_{32}^T J'_{322}\Xi_{32} + \Xi_{42}^T J'_{422}\Xi_{42} + \Xi_{52}^T J'_{522}\Xi_{52} = \sum_{\mu\nu} \Xi_{\mu 2}^T J'_{\mu 22}\Xi_{\mu 2} \end{aligned} \tag{3.71}$$

Also, $\Xi^T J' d = \begin{pmatrix} \sum \Xi_{\mu 1}^T J'_{\mu 2\nu} d_{\mu\nu} \\ \sum \Xi_{\mu 2}^T J'_{\mu 2\nu} d_{\mu\nu} \end{pmatrix} =$

$$\begin{pmatrix} \Xi_{11}^T J'_{111} d_{11} + \Xi_{21}^T J'_{211} d_{21} + \Xi_{21}^T J'_{212} d_{22} + \Xi_{31}^T J'_{311} d_{31} \\ \qquad + \Xi_{31}^T J'_{312} d_{32} + \Xi_{51}^T J'_{511} d_{51} + \Xi_{51}^T J'_{512} d_{52} \\ \Xi_{22}^T J'_{221} d_{21} + \Xi_{22}^T J'_{222} d_{22} + \Xi_{32}^T J'_{321} d_{31} + \Xi_{32}^T J'_{322} d_{32} \\ \qquad + \Xi_{42}^T J'_{422} d_{42} + \Xi_{52}^T J'_{521} d_{51} + \Xi_{52}^T J'_{522} d_{52} \end{pmatrix}. \tag{3.72}$$

Finally the parameters are found from $a = \left(\Xi^{\mathbf{T}}\mathbf{J}'\Xi\right)^{-1}\Xi^{\mathbf{T}}\mathbf{J}'d$, where $\Xi^{\mathbf{T}}\mathbf{J}'\Xi$ is a (12 $\times$ 12) matrix and $\Xi^{\mathbf{T}}\mathbf{J}'d$ is a 12 element vector. Therefore a is a 12 element vector, i.e. 6 parameters for each plate.

From (3.51) the equatorial coordinate corrections are $\beta = s\mathbf{L}^{-1}\mathbf{B}^{\mathbf{T}}\sigma_{\mathbf{x}}^{-1}(d - \Xi a)$

$$= \mathrm{s}\begin{pmatrix} \mathbf{t_{11}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{t_{21}} & \mathbf{t_{22}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{t_{31}} & \mathbf{t_{32}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{t_{42}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{t_{51}} & \mathbf{t_{52}} \end{pmatrix} \begin{pmatrix} d_{11} - \Xi_{11}a_1 \\ d_{21} - \Xi_{21}a_1 \\ d_{22} - \Xi_{22}a_2 \\ d_{31} - \Xi_{31}a_1 \\ d_{32} - \Xi_{32}a_2 \\ d_{42} - \Xi_{42}a_2 \\ d_{51} - \Xi_{51}a_1 \\ d_{52} - \Xi_{52}a_2 \end{pmatrix} \tag{3.73}$$

where $\mathbf{t}_{ij} = \mathbf{L}_{\mathrm{ii}}^{-1}\mathbf{B}_{\mathrm{ij}}^{\mathbf{T}}\sigma_{\mathrm{ij}}$ therefore:

$$\beta = s\begin{pmatrix} \mathbf{L_{11}B_{11}^T}\boldsymbol{\sigma}_{\mathbf{11}}(d_{11} - \Xi_{11}a_{11}) \\ \mathbf{L_{22}B_{21}^T}\boldsymbol{\sigma}_{\mathbf{21}}(d_{21} - \Xi_{21}a_{21}) + \mathbf{L_{22}B_{22}^T}\boldsymbol{\sigma}_{\mathbf{22}}(d_{22} - \Xi_{22}a_{22}) \\ \mathbf{L_{33}B_{31}^T}\boldsymbol{\sigma}_{\mathbf{31}}(d_{31} - \Xi_{31}a_{31}) + \mathbf{L_{33}B_{32}^T}\boldsymbol{\sigma}_{\mathbf{32}}(d_{32} - \Xi_{32}a_{32}) \\ \mathbf{L_{42}B_{42}^T}\boldsymbol{\sigma}_{\mathbf{42}}(d_{42} - \Xi_{42}a_{42}) \\ \mathbf{L_{51}B_{51}^T}\boldsymbol{\sigma}_{\mathbf{51}}(d_{51} - \Xi_{51}a_{51}) + \mathbf{L_{52}B_{52}^T}\boldsymbol{\sigma}_{\mathbf{52}}(d_{52} - \Xi_{52}a_{52}) \end{pmatrix} \tag{3.74}$$

are the ten weighted mean corrections to the star's equatorial coordinates.

Solution with Globally Constrained Parameters

This covers the essence of the theoretical analysis of the overlap solution. In this investigation the number of observations exceeds 30,000 on up to 250 plates per epoch. This many observations and plates will lead to a matrix that is very difficult to invert, purely as a result of the numerical computation loss. An alternative is to use a different method for finding the solution such as Gaussian elimination. However, if we do not invert the matrix of condition equations the covariance matrix, and hence the parameter errors, cannot be examined. This will restrict methods for checking the solution. Techniques need to be examined to reduce the size of the problem without

an appreciable loss in accuracy or flexibility. One method is to restrict the number of the parameters because the matrix of condition equations in the overlap is square with dimension "parameters times plates".

In this investigation we exposed the McCormick plates over a relatively short period, one year for the first epoch and five months for the second epoch. Over this short a period we can assume that certain parameters that characterize telescope properties will remain constant from plate to plate. (This carries the proviso that over the observation period the telescope underwent no major maintenance or repairs, for example a cleaning of the lens.) This allows us to restrict the overlap reduction to finding parameters that do not vary from plate to plate separately from those that do.

The solution changes only in the form of the matrix Ξ. In the general solution, the ν condition equations are

$$H = \left(\begin{pmatrix} x_{\mu_{\nu m_\nu}\nu} - s\xi_{\mu_{\nu m_\nu}\nu} \\ \\ y_{\mu_{\nu m_\nu}\nu} - s\eta_{\mu_{\nu m_\nu}\nu} \end{pmatrix} - \Xi_{\mu_{\nu m_\nu}\nu} a_\nu \right) = 0 \tag{3.75}$$

where for an eight constant model and the νth star

$$\Xi = \begin{pmatrix} \xi & \eta & 1 & 0 & 0 & 0 & \xi^2 & \xi\eta \\ 0 & 0 & 0 & \xi & \eta & 1 & \xi\eta & \eta^2 \end{pmatrix} \quad a = \begin{pmatrix} a \\ b \\ c \\ a' \\ b' \\ c' \\ p \\ q \end{pmatrix}. \tag{3.76}$$

Assume the tangential point parameters remain the same across the plates, e.g, p and q remain constant for all plates. In this case the relationships (3.75) will have the form

$$H = \left(\begin{pmatrix} x_{\mu_{\nu m_\nu}\nu} - s\xi_{\mu_{\nu m_\nu}\nu} \\ \\ y_{\mu_{\nu m_\nu}\nu} - s\eta_{\mu_{\nu m_\nu}\nu} \end{pmatrix} - \Xi_{\mu_{\nu m_\nu}\nu} a_\nu - \Xi_{\mu_{\nu m_\nu}\nu}' a' \right) = 0 \tag{3.77}$$

where for the νth star

$$\Xi = \begin{pmatrix} \xi & \eta & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi & \eta & 1 \end{pmatrix} \quad \Xi' = \begin{pmatrix} \xi^2 & \xi\eta \\ \xi\eta & \eta^2 \end{pmatrix}$$
$$a_\nu = \begin{pmatrix} a \\ b \\ c \\ a' \\ b' \\ c' \end{pmatrix} \quad a' = \begin{pmatrix} C_p \\ C_q \end{pmatrix}. \tag{3.78}$$

Consider the form of the matrix A given by (3.36) the three terms $\frac{\partial H}{\partial \beta}, \frac{\partial G}{\partial \beta}, \frac{\partial G}{\partial a}$ will remain as given before. The partial derivative of the condition equations H=0 with respect to the plate parameters, $\frac{\partial H}{\partial a}$, will have a different form; in the general case,

$$\begin{aligned} \frac{\partial H}{\partial a} = \frac{\partial H_\nu}{\partial(a_1 a_2 \ \cdots \ a_n)} &= \begin{pmatrix} \frac{\partial H_1}{\partial a_1} & \frac{\partial H_2}{\partial a_1} & \cdots & \frac{\partial H_\nu}{\partial a_1} \\ \frac{\partial H_1}{\partial a_2} & \frac{\partial H_2}{\partial a_2} & \cdots & \frac{\partial H_\nu}{\partial a_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial H_1}{\partial a_\nu} & \frac{\partial H_\nu}{\partial a_\nu} & \cdots & \frac{\partial H_\nu}{\partial a_\nu} \end{pmatrix} \\ &= \begin{pmatrix} \Xi_1 & 0 & \dots & 0 \\ 0 & \Xi_2 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & \Xi_n \end{pmatrix}. \end{aligned} \tag{3.79}$$

In the restricted case,

$$\begin{aligned} \frac{\partial H}{\partial a} = \frac{\partial H}{\partial(a_1 a_2 \ \cdots \ a_n a')} &= \begin{pmatrix} \frac{\partial H_1}{\partial a_1} & \frac{\partial H_1}{\partial a_2} & \cdots & \frac{\partial H_1}{\partial a_n} & \frac{\partial H_1}{\partial a'} \\ \frac{\partial H_2}{\partial a_1} & \frac{\partial H_2}{\partial a_2} & \cdots & \frac{\partial H_2}{\partial a_n} & \frac{\partial H_2}{\partial a'} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{\partial H_n}{\partial a_1} & \frac{\partial H_n}{\partial a_2} & \cdots & \frac{\partial H_n}{\partial a_n} & \frac{\partial H_n}{\partial a'} \end{pmatrix} \\ &= \begin{pmatrix} \Xi_1 & 0 & \dots & 0 & \Xi'_1 \\ 0 & \Xi_2 & \dots & 0 & \Xi'_2 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & \Xi_n & \Xi'_n \end{pmatrix}, \end{aligned} \tag{3.80}$$

where Ξ and Ξ' are as above.

The form of the development will be the same as in the generalized overlap discussion. The corrections to the reference stars will be given by,

$$\beta = s\mathbf{L}^{-1}\mathbf{B}^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}(d - \Xi a) \tag{3.81}$$

and the parameters by

$$a = (\Xi^{\mathrm{T}}\mathbf{J}'\Xi)^{-1}\Xi^{\mathrm{T}}\mathbf{J}'d \tag{3.82}$$

where the matrix Ξ is as in (3.78).

This restricted approach can be applied to many of the parameters. An initial single plate and overlap reduction will be carried out. Those parameters that have a standard deviation smaller than their error will be set constant. Allowing these parameters to vary adds nothing to the solution because they vary less than the noise of the solution, and actually make the solution worse because of computational numerical loss. A still further extension of the overlap would be to stochastically constrain the parameters that follow some known distribution (for example a normal distribution). This was developed by Eichhorn (1978), but it is beyond the scope of this investigation to apply these constraints.

Example

Consider a twelve constant overlap of the two plates, set up as in the unconstrained solution. Make the restriction that the tangential point corrections are the same for both plates. The derivation will proceed as before, with these changes

$$F_{\mu_\nu\nu} = \left(\begin{pmatrix} x_{\mu_\nu\nu} - s\xi_{\mu_\nu\nu} \\ y_{\mu_\nu\nu} - s\eta_{\mu_\nu\nu} \end{pmatrix} - \Xi_{\mu_\nu\nu}a_\nu - \Xi'_{\mu_\nu\nu}a' \right), \tag{3.83}$$

where the matrix Ξ is a (16 $\times$ 22) matrix of the form

$$\Xi^{\mathrm{T}} = \begin{pmatrix} \Xi^{\mathrm{T}}_{11} & \Xi_{21} & 0 & \Xi^{\mathrm{T}}_{31} & 0 & 0 & \Xi^{\mathrm{T}}_{51} & 0 \\ 0 & 0 & \Xi^{\mathrm{T}}_{22} & 0 & \Xi^{\mathrm{T}}_{32} & \Xi^{\mathrm{T}}_{42} & 0 & \Xi^{\mathrm{T}}_{52} \\ \Xi^{\mathrm{T}\prime}_{11} & \Xi^{\mathrm{T}\prime}_{21} & \Xi^{\mathrm{T}\prime}_{22} & \Xi^{\mathrm{T}\prime}_{31} & \Xi^{\mathrm{T}\prime}_{32} & \Xi^{\mathrm{T}\prime}_{42} & \Xi^{\mathrm{T}\prime}_{51} & \Xi^{\mathrm{T}\prime}_{52} \end{pmatrix}. \tag{3.84}$$

Each $\Xi_{\mu\nu}$ is a (2 $\times$ 10) matrix, thus

$$\Xi_{\mu\nu} = \begin{pmatrix} \xi_{\mu\nu} & \eta_{\mu\nu} & 1 & 0 & 0 & 0 & m_{\mu\nu} - m_o & 0 & m_{\mu\nu} - m_o\xi_{\mu\nu} & \xi_{\mu\nu}(\xi^2_{\mu\nu} + \eta^2_{\mu\nu}) \\ 0 & 0 & 0 & \xi_{\mu\nu} & \eta_{\mu\nu} & 1 & 0 & m_{\mu\nu} - m_o & m_{\mu\nu} - m_o\eta_{\mu\nu} & \eta_{\mu\nu}(\xi^2_{\mu\nu} + \eta^2_{\mu\nu}) \end{pmatrix}. \tag{3.85}$$

$\Xi'_{\mu\nu}$ is a (2×2) matrix given by

$$\Xi'_{\mu\nu} = \begin{pmatrix} \xi^2_{\mu\nu} & \xi_{\mu\nu}\eta_{\mu\nu} \\ \xi_{\mu\nu}\eta_{\mu\nu} & \eta^2_{\mu\nu} \end{pmatrix}. \tag{3.86}$$

From (3.51) the corrections to the stellar coordinates will be

$$\beta = s\mathbf{L}^{-1}\mathbf{B}^{\mathrm{T}}\sigma_{\mathbf{x}}^{-1}(d - \Xi a) \tag{3.87}$$

where the matrix Ξ is as above and the vector a will be of the form

$$a = \begin{pmatrix} a_1 \\ a_2 \\ a' \end{pmatrix} \quad \text{where} \quad a_\nu = \begin{pmatrix} a \\ b \\ c \\ a' \\ b' \\ c' \\ e \\ f \\ g \\ h \end{pmatrix} \quad a' = \begin{pmatrix} C_p \\ C_q \end{pmatrix}. \tag{3.88}$$

For the parameters $a = \begin{pmatrix} a_1 \\ a_2 \\ a' \end{pmatrix} \to (22 \times 1) = \begin{pmatrix} (10 \times 1) \\ (10 \times 1) \\ (2 \times 1) \end{pmatrix}$ and given by $\left(\Xi^{\mathrm{T}}\mathbf{J}'\Xi\right)^{-1}\Xi^{\mathrm{T}}\mathbf{J}'d \to ((22 \times 16)(16 \times 16)(16 \times 22))^{-1}(22 \times 16)(16 \times 16)(16 \times 1) = (22 \times 1)$.

Using J from equation (3.67), we get

$$\Xi^{\mathrm{T}}\mathbf{J}'\Xi =$$

$$\begin{pmatrix} \Xi^{\mathrm{T}}_{11} & \Xi_{21} & 0 & \Xi^{\mathrm{T}}_{31} & 0 & 0 & \Xi^{\mathrm{T}}_{51} & 0 \\ 0 & 0 & \Xi^{\mathrm{T}}_{22} & 0 & \Xi^{\mathrm{T}}_{32} & \Xi^{\mathrm{T}}_{42} & 0 & \Xi^{\mathrm{T}}_{52} \\ \Xi^{\mathrm{T}\prime}_{11} & \Xi^{\mathrm{T}\prime}_{21} & \Xi^{\mathrm{T}\prime}_{22} & \Xi^{\mathrm{T}\prime}_{31} & \Xi^{\mathrm{T}\prime}_{32} & \Xi^{\mathrm{T}\prime}_{42} & \Xi^{\mathrm{T}\prime}_{51} & \Xi^{\mathrm{T}\prime}_{52} \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{J}'_{111} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{J}'_{211} & \mathbf{J}'_{212} & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{J}'_{221} & \mathbf{J}'_{222} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{J}'_{311} & \mathbf{J}'_{312} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{J}'_{321} & \mathbf{J}'_{322} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{J}'_{422} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{J}'_{511} & \mathbf{J}'_{512} \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{J}'_{521} & \mathbf{J}'_{522} \end{pmatrix} \begin{pmatrix} \Xi_{11} & 0 & \Xi_{11}' \\ \Xi_{21} & 0 & \Xi_{21}' \\ 0 & \Xi_{22} & \Xi_{22}' \\ \Xi_{31} & 0 & \Xi_{31}' \\ 0 & \Xi_{32} & \Xi_{32}' \\ 0 & \Xi_{42} & \Xi_{42}' \\ \Xi_{51} & 0 & \Xi_{51}' \\ 0 & \Xi_{52} & \Xi_{52}' \end{pmatrix} \tag{3.89}$$

and

$$\Xi^{T}J'\Xi =$$
$$\begin{pmatrix} \Xi_{11}^{T} & \Xi_{21}^{T} & 0 & \Xi_{31}^{T} & 0 & 0 & \Xi_{51}^{T} & 0 \\ 0 & 0 & \Xi_{22}^{T} & 0 & \Xi_{32}^{T} & \Xi_{42}^{T} & 0 & \Xi_{52}^{T} \\ \Xi_{11}^{T\prime} & \Xi_{21}^{T\prime} & \Xi_{22}^{T\prime} & \Xi_{31}^{T\prime} & \Xi_{32}^{T\prime} & \Xi_{42}^{T\prime} & \Xi_{51}^{T\prime} & \Xi_{52}^{T\prime} \end{pmatrix}$$

$$\begin{pmatrix} J'_{111}\Xi_{11} & 0 & J'_{111}\Xi_{11}' \\ J'_{211}\Xi_{21} & J'_{212}\Xi_{22} & J'_{211}\Xi_{21}' + J'_{212}\Xi_{22}' \\ J'_{221}\Xi_{21} & J'_{222}\Xi_{22} & J'_{221}\Xi_{21}' + J'_{222}\Xi_{22}' \\ J'_{311}\Xi_{31} & J'_{312}\Xi_{32} & J'_{311}\Xi_{31}' + J'_{312}\Xi_{32}' \\ J'_{321}\Xi_{31} & J'_{322}\Xi_{32} & J'_{321}\Xi_{31}' + J'_{322}\Xi_{32}' \\ 0 & J'_{422}\Xi_{42} & J'_{422}\Xi_{42}' \\ J'_{511}\Xi_{51} & J'_{512}\Xi_{52} & J'_{511}\Xi_{51}' + J'_{512}\Xi_{52}' \\ J'_{521}\Xi_{51} & J'_{522}\Xi_{52} & J'_{522}\Xi_{52}' + J'_{521}\Xi_{51}' \end{pmatrix}. \tag{3.90}$$

Therefore

$$\Xi^{T}J\Xi = \begin{pmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ \Pi_{21} & \Pi_{22} & \Pi_{23} \\ \Pi_{31} & \Pi_{32} & \Pi_{33} \end{pmatrix} \tag{3.91}$$

where

$$\begin{aligned} \Pi_{11} &= \Xi_{11}^{T}J'_{111}\Xi_{11} + \Xi_{21}^{T}J'_{211}\Xi_{21} + \Xi_{31}^{T}J'_{311}\Xi_{31} + \Xi_{51}^{T}J'_{511}\Xi_{51} \\ \Pi_{12} &= \Xi_{21}^{T}J'_{212}\Xi_{22} + \Xi_{31}^{T}J'_{312}\Xi_{32} + \Xi_{51}^{T}J'_{512}\Xi_{52} \\ \Pi_{13} &= \Xi_{11}^{T}J'_{111}\Xi_{11}' + \Xi_{21}^{T}J'_{211}\Xi_{21}' + \Xi_{21}^{T}J'_{212}\Xi_{22}' + \Xi_{31}^{T}J'_{311}\Xi_{31}' \\ &\quad + \Xi_{31}^{T}J'_{312}\Xi_{32}' + \Xi_{51}^{T}J'_{511}\Xi_{51}' + \Xi_{51}^{T}J'_{512}\Xi_{52}' \end{aligned} \tag{3.92}$$

$$\begin{aligned} \Pi_{21} &= \Xi_{22}^{T}J'_{221}\Xi_{21} + \Xi_{32}^{T}J'_{321}\Xi_{31} + \Xi_{52}^{T}J'_{521} \\ \Pi_{22} &= \Xi_{22}^{T}J'_{222}\Xi_{22} + \Xi_{32}^{T}J'_{322}\Xi_{32} + \Xi_{42}^{T}J'_{422}\Xi_{42} + \Xi_{52}^{T}J'_{522}\Xi_{52} \\ \Pi_{23} &= \Xi_{22}^{T}(J'_{221}\Xi_{21}' + J'_{222}\Xi_{22}') + \Xi_{32}^{T}(J'_{321}\Xi_{31}' + J'_{322}\Xi_{32}') \\ &\quad + \Xi_{42}^{T}(J'_{422}\Xi_{42}') + \Xi_{52}^{T}(J'_{522}\Xi_{52}' + J'_{521}\Xi_{51}') \end{aligned} \tag{3.93}$$

$$
\begin{aligned}
\Pi_{31} = {} & \Xi_{11}^{T}(J'_{111}\Xi_{11}) + \Xi_{21}^{T}(J'_{221}\Xi_{21}) + \Xi_{22}^{T}(J'_{221}\Xi_{21}) + \Xi_{31}^{T}(J'_{311}\Xi_{31}) \\
& + \Xi_{32}^{T}(J'_{321}\Xi_{31}) + \Xi_{51}^{T}(J'_{511}\Xi_{51}) + \Xi_{52}^{T}(J'_{521}\Xi_{51}) \\
\Pi_{32} = {} & \Xi_{21}^{T}(J'_{212}\Xi_{22}) + \Xi_{22}^{T}(J'_{222}\Xi_{22}) + \Xi_{31}^{T}(J'_{312}\Xi_{32}) + \Xi_{32}^{T}(J'_{322}\Xi_{32}) \\
& + \Xi_{42}^{T}(J'_{422}\Xi_{42}) + \Xi_{51}^{T}(J'_{512}\Xi_{52}) + \Xi_{52}^{T}(J'_{522}\Xi_{52}) \\
\Pi_{33} = {} & \Xi_{31}^{T}(J'_{311}\Xi_{31}' + J'_{312}\Xi_{32}') + \Xi_{32}^{T}(J'_{321}\Xi_{31}' + J'_{322}\Xi_{32}') \\
& + \Xi_{42}^{T}(J'_{422}\Xi_{42}') + \Xi_{51}^{T}(J'_{511}\Xi_{51}' + J'_{512}\Xi_{52}') + \Xi_{52}^{T}(J'_{522}\Xi_{52}' + J'_{521}\Xi_{51}'),
\end{aligned}
\tag{3.94}
$$

or in summation format

$$
\Xi^{T} J' \Xi = \begin{pmatrix}
\sum\limits_{\mu\nu} \Xi_{\mu 1}^{T} J'_{\mu 11} \Xi_{\mu 1} & \sum\limits_{\mu\nu} \Xi_{\mu 2}^{T} J'_{\mu 21} \Xi_{\mu 1} & \sum\limits_{\mu\nu} \Xi_{\mu\nu}^{T} J'_{\mu\nu 1} \Xi_{\mu\nu}' \\
\sum\limits_{\mu\nu} \Xi_{\mu 1}^{T} J'_{\mu 12} \Xi_{\mu 2} & \sum\limits_{\mu\nu} \Xi_{\mu 2}^{T} J'_{\mu 22} \Xi_{\mu 2} & \sum\limits_{\mu\nu} \Xi_{\mu\nu}^{T} J'_{\mu\nu 2} \Xi_{\mu\nu}' \\
\sum\limits_{\mu\nu} \Xi_{\mu\nu}^{T} J'_{\mu\nu 1} \Xi_{\mu\nu} & \sum\limits_{\mu\nu} \Xi_{\mu\nu}^{T} J'_{\mu\nu 2} \Xi_{\mu\nu} & \sum\limits_{\mu\nu} \Xi_{\mu\nu}^{T} J'_{\mu\nu\nu} \Xi_{\mu\nu}'
\end{pmatrix}.
\tag{3.95}
$$

The term: $\Xi^{T} J' d$

$$
= \begin{pmatrix}
\Xi_{11}^{T} & \Xi_{21} & 0 & \Xi_{31}^{T} & 0 & 0 & \Xi_{51}^{T} & 0 \\
0 & 0 & \Xi_{22}^{T} & 0 & \Xi_{32}^{T} & \Xi_{42}^{T} & 0 & \Xi_{52}^{T} \\
\Xi_{11}^{T\prime} & \Xi_{21}^{T\prime} & \Xi_{22}^{T\prime} & \Xi_{31}^{T\prime} & \Xi_{32}^{T\prime} & \Xi_{42}^{T\prime} & \Xi_{51}^{T\prime} & \Xi_{52}^{T\prime}
\end{pmatrix}
\begin{pmatrix}
J'_{111} d_{11} \\
J'_{211} d_{21} + J'_{212} d_{22} \\
J'_{221} d_{21} + J'_{222} d_{22} \\
J'_{311} d_{31} + J'_{312} d_{32} \\
J'_{321} d_{31} + J'_{322} d_{32} \\
J'_{422} d_{42} \\
J'_{511} d_{51} + J'_{512} d_{52} \\
J'_{521} d_{51} + J'_{522} d_{52}
\end{pmatrix}
$$

$$
= \begin{pmatrix}
\begin{gathered}
\Xi_{11}^{T} J'_{111} d_{11} + \Xi_{21}(J'_{211} d_{21} + J'_{212} d_{22}) + \Xi_{31}^{T}(J'_{311} d_{31} + J'_{312} d_{32}) \\
+ \Xi_{51}^{T}(J'_{511} d_{51} + J'_{512} d_{52})
\end{gathered} \\ \\
\begin{gathered}
\Xi_{22}^{T}(J'_{221} d_{21} + J'_{222} d_{22}) + \Xi_{32}^{T}(J'_{321} d_{31} + J'_{322} d_{32}) + \Xi_{42}^{T}(J'_{422} d_{42}) \\
+ \Xi_{52}^{T}(J'_{521} d_{51} + J'_{522} d_{52})
\end{gathered} \\ \\
\begin{gathered}
\Xi_{11}^{T} J'_{111} d_{11} + \Xi_{21}^{T}(J'_{211} d_{21} + J'_{212} d_{22}) + \Xi_{22}^{T}(J'_{221} d_{21} + J'_{222} d_{22}) \\
+ \Xi_{31}^{T}(J'_{311} d_{31} + J'_{312} d_{32}) + \Xi_{32}^{T}(J'_{321} d_{31} + J'_{322} d_{32}) + \Xi_{42}^{T\prime} \\
(J'_{422} d_{42}) + \Xi_{51}^{T}(J'_{511} d_{51} + J'_{512} d_{52}) + \Xi_{52}^{T}(J'_{521} d_{51} + J'_{522} d_{52})
\end{gathered}
\end{pmatrix}
\tag{3.96}
$$

which can be written as

$$\Xi^{\mathbf{T}}\mathbf{J}'d = \begin{pmatrix} \sum \Xi^{\mathbf{T}}_{\mu 1}\mathbf{J}'_{\mu 1\nu}\mathbf{d}_{\mu\nu} \\ \sum \Xi^{\mathbf{T}}_{\mu 2}\mathbf{J}'_{\mu 2\nu}\mathbf{d}_{\mu\nu} \\ \sum \Xi^{\mathbf{T}}_{\mu\nu}\mathbf{J}'_{\mu\nu\nu}\mathbf{d}_{\mu\nu} \end{pmatrix}. \tag{3.97}$$

With these products we can find the parameters and the corrections to the stellar coordinates.

Theoretical Plate Modelling Terms

In the single and overlap development we use the example of a six constant model. In reality a more sophisticated model is needed to account accurately for the complex image formed by a telescope. This section discusses the various terms and their physical meaning.

Assume that a telescope has the same projection properties as a pinhole camera as shown in figure 3. In this ideal system the measured coordinates (x,y) with origin at the tangential point T would be related to the standard coordinates (ξ,η) by only a factor of the focal length of the telescope. In reality there are many deviations from the ideal situation. Below is a list of some of the more common ones. (This list is not exhaustive.)

- (ξ,η) axes rotated against the (x,y) axes
- origin of the (x,y) frame not at the tangential point
- tilt of the (ξ,η) focal plane w.r.t the (x,y) plate plane and noncoincidence of the optical axis with the origin of the (ξ,η) coordinate frame
- magnitude and coma effects
- radial distortion
- tangential distortion due to components of objective being improperly aligned

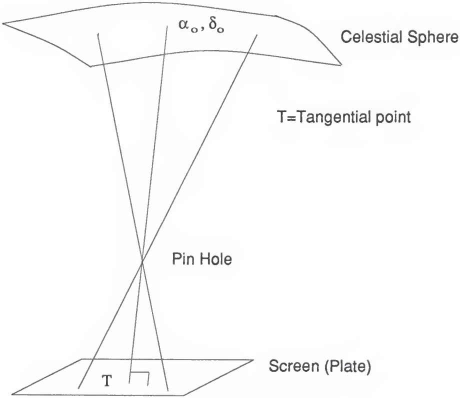


Figure 3: Projection of a Pinhole Camera

- different scales on each axis due to different measuring screws, and/or refraction of the atmosphere
- nonorthogonality of the x and y axes
- guiding errors and developing errors (e.g., emulsion shift)
- color terms

These deviations are not independent of each other, and thus there is a significant crossover of effects. The second order effects are often negligible and mixed into first order effects. It is useful to discuss the theoretical form of the more important deviations we model in this investigation.

Departures from a Gnomonic Projection

Noncoincidence of axes

The positioning of the photographic plate in the telescope and then within the measuring machine will lead to a rotation of the plate with respect to the frame of reference on the sky. This means the (ξ,η) axis, defining the frame of the sky, will be rotated with respect to the (x,y) axis, defining the frame of the measuring machine. Noncoincidence of axes A rotation of the (ξ,η) frame will correct this:

$$\begin{pmatrix} x \\ y \end{pmatrix} = s \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \xi \\ \eta \end{pmatrix} \tag{3.98}$$

where s is the focal length.

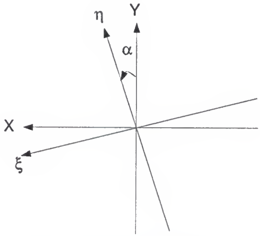


Figure 4: Noncoincidence of axes

Origin shift

In the process of transference from the telescope to the measuring machine there will be, in addition to a plate rotation, a shift in the origin of the (ξ,η) coordinate frame

with respect to the (x,y) frame. If the origin of the (ξ,η) frame in the (x,y) frame is (c,c′), then this can be allowed by a constant subtraction from the (x,y) coordinates. Combining this change into equation (3.98) yields:

$$\begin{pmatrix} x-c \\ y-c' \end{pmatrix} = s\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \xi \\ \eta \end{pmatrix}. \tag{3.99}$$

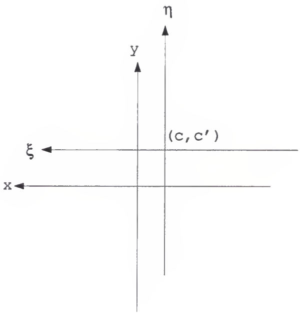


Figure 5: Origin Shift

Incorrect scale

In an ideal system the (ξ,η) coordinates would be related to the (x,y) coordinates by the focal length; i.e. $(x,y) = s\ (\xi,\eta)$. In practice the actual focal length of the telescope will vary due to temperature changes and everyday use. We therefore treat this as a variable and correct by incorporating a small modification in the direct ξ and η terms.

The relationship between standard and rectangular coordinates now has the form

$$\begin{pmatrix} x-c \\ y-c' \end{pmatrix} = s\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \xi \\ \eta \end{pmatrix} - \begin{pmatrix} a \\ a' \end{pmatrix}\begin{pmatrix} \xi \\ \eta \end{pmatrix} \tag{3.100}$$

where a and a' are constants.

These terms can be combined if we include the cosine and sine terms as constants, then the deviations; noncoincidence, origin shift and scale difference in matrix format are

$$\begin{aligned} \begin{pmatrix} x-c \\ y-c' \end{pmatrix} &= s\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \xi \\ \eta \end{pmatrix} - \begin{pmatrix} a \\ a' \end{pmatrix}\begin{pmatrix} \xi \\ \eta \end{pmatrix} \\ \rightarrow \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} s\xi\cos\alpha + s\eta\sin\alpha - a\xi + c \\ -s\xi\sin\alpha + s\eta\cos\alpha - a'\eta + c' \end{pmatrix} \\ &= s\begin{pmatrix} (\cos\alpha - \frac{a}{s})\xi + (\sin\alpha)\eta + \frac{c}{s} \\ \left(\cos\alpha - \frac{a'}{s}\right)\eta - (\sin\alpha)\xi + \frac{c'}{s} \end{pmatrix} \\ \begin{pmatrix} x \\ y \end{pmatrix} &= s\begin{pmatrix} \xi & \eta & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi & \eta & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ a' \\ b' \\ c' \end{pmatrix} \end{aligned} \tag{3.101}$$

where $a = \cos\alpha - \frac{a}{s}$, $b = \sin\alpha$ $c = \frac{c}{s}$, $a' = -\sin\alpha$, $b' = \cos\alpha - \frac{a'}{s}$, and $c' = \frac{c'}{s}$.

This is the **six constant model**. The rest of this discussion will build on this model; it is therefore important to first say a word about this idea of modelling. From equation (3.101) it would appear that if our projection were only subject to the deviations: non-conjunction of the axis and scale differences, then b=-a'. However this does not happen even in this simple case. This is because the above analysis hides second order effects, and we have made no provisions to account for atmospheric refraction. There will be second order mixing and the reader should be aware that these physical interpretations of the models are only good to the first order. This will be alleviated as we add more terms that are more difficult to model.

Focal plane tilt and an incorrect tangential point

If either the photographic plate is tilted with respect to the focal plane or the assumed tangential point is not the same as the intersection of the optical axis with the plate, the differential change in (x,y) with respect to the plate center (α_o,δ_o) is

$$\begin{aligned} \frac{\partial(x,y)}{\partial(\alpha_0,\delta_0)} &= \frac{\partial(x,y)}{\partial(\xi,\eta)}\frac{\partial(\xi,\eta)}{\partial(\alpha_0,\delta_0)} \\ \frac{\partial(x,y)}{\partial(\xi,\eta)} &= \begin{pmatrix} a & b \\ a' & b' \end{pmatrix} \\ \frac{\partial(\xi,\eta)}{\partial(\alpha_0,\delta_0)} &= \frac{\partial(\xi,\eta)}{\partial(\Xi,\mathrm{H},\mathrm{Z})}\frac{\partial(\Xi,\mathrm{H},\mathrm{Z})}{\partial(\alpha_0,\delta_0)}. \end{aligned} \tag{3.102}$$

From equation (3.2) we get

$$\begin{aligned} &\xi = \frac{\Xi}{\mathrm{Z}},\quad \eta = \frac{\mathrm{H}}{\mathrm{Z}} \\ &\rightarrow \frac{\partial(\xi,\eta)}{\partial(\Xi,\mathrm{H},\mathrm{Z})} = \frac{1}{\mathrm{Z}}\begin{pmatrix} 1 & 0 & -\xi \\ 0 & 1 & -\eta \end{pmatrix} \end{aligned} \tag{3.103}$$

and

$$\begin{aligned} \Upsilon = \begin{pmatrix} \Xi \\ \mathrm{H} \\ \mathrm{Z} \end{pmatrix} &= \begin{pmatrix} \cos\delta\sin(\alpha-\alpha_0) \\ \cos\delta_0\sin\delta - \sin\delta_0\cos\delta\cos(\alpha-\alpha_0) \\ \sin\delta_0\sin\delta + \cos\delta_0\cos\delta\cos(\alpha-\alpha_0) \end{pmatrix} \\ \frac{\partial(\Xi,\mathrm{H},\mathrm{Z})}{\partial(\alpha_0,\delta_0)} &= \begin{pmatrix} -\cos\delta\cos(\alpha-\alpha_0) & 0 \\ -\sin\delta_0\cos\delta\sin(\alpha-\alpha_0) & -\sin\delta_0\sin\delta - \cos\delta_0\cos\delta\cos(\alpha-\alpha_0) \\ \cos\delta_0\cos\delta\sin(\alpha-\alpha_0) & \sin\delta_0\cos\delta - \sin\delta_0\cos\delta\cos(\alpha-\alpha_0) \end{pmatrix} \\ \frac{\partial(\Xi,\mathrm{H},\mathrm{Z})}{\partial(\alpha_0,\delta_0)} &= \begin{pmatrix} \mathrm{H}\sin\delta_0 - \mathrm{Z}\cos\delta_0 & 0 \\ -\Xi\sin\delta_0 & \mathrm{Z} \\ \mathrm{H}\cos\delta_0 & \mathrm{H} \end{pmatrix}. \end{aligned} \tag{3.104}$$

This simplifies to

$$\begin{aligned} \frac{\partial(\xi,\eta)}{\partial(\alpha_0,\delta_0)} &= \frac{\partial(\xi,\eta)}{\partial(\Xi,\mathrm{H},\mathrm{Z})}\frac{\partial(\Xi,\mathrm{H},\mathrm{Z})}{\partial(\alpha_0,\delta_0)} \\ &= \frac{1}{\mathrm{Z}}\begin{pmatrix} 1 & 0 & -\xi \\ 0 & 1 & -\eta \end{pmatrix}\begin{pmatrix} \mathrm{H}\sin\delta_0 - \mathrm{Z}\cos\delta_0 & 0 \\ -\Xi\sin\delta_0 & \mathrm{Z} \\ \Xi\cos\delta_0 & \mathrm{H} \end{pmatrix} \\ &= \begin{pmatrix} \eta\sin\delta_0 - \cos\delta_0 - \xi^2\cos\delta_0 & -\xi \\ -\xi\sin\delta_0 - \xi\eta\cos\delta_0 & -1-\eta^2 \end{pmatrix} \end{aligned} \tag{3.105}$$

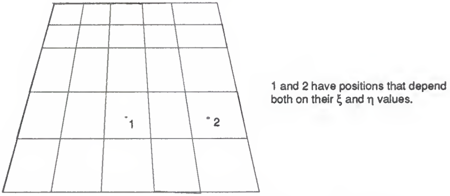


Figure 6: Effect of Plate Tilt or Incorrect Tangential Point

and finally,

$$
\begin{aligned}
\frac{\partial(x,y)}{\partial(\alpha_0,\delta_0)} &= \begin{pmatrix} a & b \\ a' & b' \end{pmatrix} \begin{pmatrix} \eta\sin\delta_0 - \cos\delta_0 - \xi^2\cos\delta_0 & -\xi \\ -\xi\sin\delta_0 - \xi\eta\cos\delta_0 & -1-\eta^2 \end{pmatrix} \\
&= \begin{pmatrix} a\left(\eta\sin\delta_0 - \cos\delta_0 - \xi^2\cos\delta_0\right) - b(-\xi\sin\delta_0 - \xi\eta\cos\delta_0) \\ -a'\xi - b'(-1-\eta^2) \end{pmatrix} \\
&= \begin{pmatrix} -a\cos\delta_0 + (b\sin\delta_0)\xi + (a\sin\delta_0)\eta + (b\cos\delta_0)\xi\eta - (a\cos\delta_0)\xi^2 \\ b' - a'\xi + b'\eta^2 \end{pmatrix}
\end{aligned}
\tag{3.106}
$$

(cf. Eichhorn, 1974).

Most models will at a minimum contain the linear six constant model. (If we have an ideal measuring machine then we can use a four constant model if the effects of differential atmospheric refraction are removed beforehand.) The constant term $-a\cos\delta_0$ and the η term $a\sin\delta_0$ in (3.106) can be incorporated in the six constant c′ and b parameters respectively. The only new terms that need to be introduced are those that are functions of $\xi\eta, \xi^2$ and η^2.

These terms, introduced by a tangential point shift, are also those terms that occur in plate (focal plane) tilt. We illustrate this by looking at the geometry. Tilting of the plate causes the coordinate system to have different scales depending on the position in the plate.

The ξ_x,η_x coordinates of the point x will be functions of the ideal (gnomonic) ξ and η respectively, e.g $\xi_x=\xi_x(\xi,\eta)$ and $\eta_x=\eta_x(\xi,\eta)$. So the tilt will add square and cross terms in ξ and η into the model. So we can allow for both these effects by introducing the terms

$$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} \xi^2 & \xi\eta \\ \xi\eta & \eta^2 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} \tag{3.107}$$

where p and q are constants.

Magnitude and coma effects

The stellar magnitude terms and lens coma will cause the image of a star to shift. This is particularly important in studies that use concentric plates as these terms will cause an expansion of the observed region. The reason is simple, a brighter star will cause the centroid found for an image to move out along a radial vector. Figure 7 shows, in a highly exaggerated example, this effect for two stars; the radial vector is pointing down.

The two images should have the same position for their centroid, but because of coma the best fit ellipse in the brighter star is centered further in along the radial vector than that for the dimmer star. We model these effects by including the terms

$$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} m-\langle m\rangle & 0 & (m-\langle m\rangle)\xi & (m-\langle m\rangle)\xi^2 \\ 0 & m-\langle m\rangle & (m-\langle m\rangle)\eta & (m-\langle m\rangle)\eta^2 \end{pmatrix} \begin{pmatrix} e \\ f \\ g \\ h \end{pmatrix} \tag{3.108}$$

where e, f, g, and h are constants.

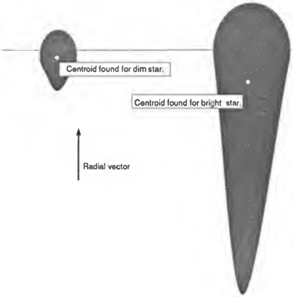


Figure 7: Effect of Magnitude and Coma Terms

Radial distortion

Barrel and pincushion distortion are lens aberrations which produce radial terms. To correct for this distortion a term of the form

$$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} \xi(\xi^2 + \eta^2) \\ \eta(\xi^2 + \eta^2) \end{pmatrix} (i) \tag{3.109}$$

where i is a constant, is included in many models.

This completes the discussion of the more important departures from a gnomonic projection that will affect the position of a star. There are many other possible systematic departures from a gnomonic projection, color terms, magnitude squared terms, radial magnitude terms to list a few. Analysis of the residuals will highlight those terms to include.

Typical Models

Assume the relationship of measured coordinates to standard coordinates is of the form $\begin{pmatrix} x \\ y \end{pmatrix} = s\begin{pmatrix} \xi \\ \eta \end{pmatrix} + \Xi a$, where Ξ is the model matrix and a is the parameter vector. Below we examine the 6, 8, 12, and 16 constant models. Other models are also tested in this study, but the ones listed are good examples of the combinations tried.

The 6 constant model

We have already discussed the 6 constant model and its physical interpretation.

$$\Xi = \begin{pmatrix} \xi & \eta & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi & \eta & 1 \end{pmatrix}, \quad a = \begin{pmatrix} a \\ b \\ c \\ a' \\ b' \\ c' \end{pmatrix} \tag{3.110}$$

- c and c′ account for differences between the origins of the measurable rectangular image coordinates (x,y) and the standard coordinates (ξ,η).
- a and b′ allow for an incorrect focal length and different x and y scales as caused by differential refraction.
- b and a′ allow for rotation of the x,y frame vs the ξ,η frame and non-perpendicularity of the axes.

The 8 constant model

For the 8 constant model we just add linear terms in the magnitude.

$$\Xi = \begin{pmatrix} \xi & \eta & 1 & 0 & 0 & 0 & m - \langle m \rangle & 0 \\ 0 & 0 & 0 & \xi & \eta & 1 & 0 & m - \langle m \rangle \end{pmatrix}, \quad a = \begin{pmatrix} a \\ b \\ c \\ a' \\ b' \\ c' \\ e \\ f \end{pmatrix} \tag{3.111}$$

- e and f allow for magnitude effects

This model is only really important to use as a base for introducing global parameters. It corrects for the major plate–to–plate dependent terms resulting from guiding, scale and plate placement errors. Additional parameters may be considered global, and should be tested for each set of plates for that possibility.

The 12 constant model

If we add in terms to correct for plate tilt, coma and, radial effects we get the 12 constant model as shown in (3.112). In this model the tilt and tangential point

corrections are assumed to have dependencies that mirror in the x and y directions.

$$\Xi = \begin{pmatrix} \xi & \eta & 1 & 0 & 0 & 0 & \xi^2 & \xi\eta & m-\langle m\rangle & 0 & (m-\langle m\rangle)\xi & \xi(\xi^2+\eta^2) \\ 0 & 0 & 0 & \xi & \eta & 1 & \xi\eta & \eta^2 & 0 & m-\langle m\rangle & (m-\langle m\rangle)\eta & \eta(\xi^2+\eta^2) \end{pmatrix}$$

$$a = \begin{pmatrix} a \\ b \\ c \\ a' \\ b' \\ c' \\ p \\ q \\ e \\ f \\ g \\ h \end{pmatrix} \tag{3.112}$$

- p and q allow for tilt and tangential point corrections
- e and f allow for magnitude effects
- g corrects for coma, it should have the same coefficient in both x an y
- h corrects for radial distortion, again the coefficient should be the same.

The 16 constant model

The most sophisticated model usually attempted is the sixteen constant model. Here we allow the tilt and tangential point corrections to have different x and y dependencies

(parameters).

$$\Xi = \begin{pmatrix} \xi & \eta & 1 & 0 & 0 & 0 & \xi^2 & \xi\eta & \eta^2 & 0 & 0 & 0 & m-\langle m\rangle & 0 & (m-\langle m\rangle)\xi & \xi(\xi^2+\eta^2) \\ 0 & 0 & 0 & \xi & \eta & 1 & 0 & 0 & 0 & \xi^2 & \xi\eta & \eta^2 & 0 & m-\langle m\rangle & (m-\langle m\rangle)\eta & \eta(\xi^2+\eta^2) \end{pmatrix}$$

$$\& \quad a = \begin{pmatrix} a \\ b \\ c \\ a' \\ b' \\ c' \\ p \\ q \\ r \\ p' \\ q' \\ r' \\ e \\ f \\ g \\ i \end{pmatrix} \tag{3.113}$$

- r, p′, q′, and r′ allow for separate tilt and tangential point corrections.

This model proved to be dominated by the cubic radial term and another form of it replaces this term with a second order magnitude modelling term, $\begin{pmatrix} (m-\langle m\rangle)\xi^2 \\ (m-\langle m\rangle)\eta^2 \end{pmatrix}(\,h\,)$.

Summary of potential model terms

The terms that might be included can be simply split into three categories. Terms that usually require different parameters in the functional forms of the x and y coordinates; terms that may have the same parameter in both the x and y values, but may also be split; and finally terms that usually have the same parameters in x and y, i.e. are radial in nature, and will not usually be split.

Terms that usually have different x and y parameters include all the linear terms in the standard coordinates and the magnitude: ξ, η, and, $(m - \langle m \rangle)$. These are strongly dependent upon the conditions of that particular exposure, e.g. centering of the plate, guiding during exposure, and scale variations (which are in turn dependent on the temperature at the time of exposure). These terms are then usually found for each plate and each coordinate on those plates; it would be unwise to try and globally fit them or constrain them to the same value for both coordinates. Because of their usually small random variation the scale terms, the direct terms in ξ and η, are very good candidates for stochastic constraints because they would usually be assumed to be normally distributed around some mean value.

Model terms that are sometimes different in the x and y parameters and sometimes constrained to the same value for both the coordinates are the second order terms (ξ^2, η^2, and $\xi\eta$) and — if they are included — cubic terms (ξ^3, η^3 $\xi^2\eta$, and, $\eta^2\xi$). These allow for plate tilt, tangential point corrections, some distortion effects and differential atmospheric refraction; they are usually small. If an analysis of their values in a single plate solution shows that they vary widely for the two coordinates, then they can be split. These parameters are very telescope dependent, and providing the telescope undergoes no overhauls during the period of exposure for a set of plates, these parameters will be good candidates for global constraints.

The distortion terms [$\xi(\xi^2{+}\eta^2)$ and $\eta(\xi^2{+}\eta^2)$] and coma terms [$(m - \langle m \rangle)\xi$, $(m - \langle m \rangle)\eta$, $(m - \langle m \rangle)\xi^2$, $(m - \langle m \rangle)\eta^2$, and $(m - \langle m \rangle)^2$] can usually be assumed to be the same in both the x and y coordinate. These terms are mainly related to the lens, and they may be considered either globally constrained, stochastically constrained

or removed in some pre-overlap process. The nature of the actual problem indicates the road to follow.

As will be discussed in the next chapter the use of many parameters may introduce large variances in the possible solution, while leaving out terms introduces the risk of not including effects that may be very important to model. An analysis of parameter variance by Eichhorn and Williams (1963) shows that including unneeded terms will increase the ultimate error of the stellar position by the introduction of errors through the parameter variances.

CHAPTER 4
OBSERVATIONS

We have obtained observations from three epochs. The first epoch (the Astrographic Catalogue, AC) is available in the form of published x and y coordinates. The second and third epochs required measuring of photographic plates on an automatic plate measuring machine. The first two sections in this chapter will discuss the observations, and the last section will discuss the measuring of the plates.

Collection of the Observations

The First Epoch

For the first epoch we used the Astrographic Catalogue (AC). The AC was one of the largest astronomical enterprises ever undertaken. In 1890 twenty-two observatories all over the world started a program to map the sky using a 'standard' astrograph. A standard astrograph is a double telescope whose photographically corrected lens has a plate scale of $1'$=1mm. The hope was that by requiring this adherence to a particular instrument and by imposing strict guidelines on the measurements of the stellar image coordinates there would be a consistent map of the whole sky for an epoch around 1900.

The AC has provided an invaluable reference source for astronomers ever since. In particular it provides a long baseline and has a strong corner-in-center overlapping plate pattern. The observation of the sky was finally completed in 1950, and the measuring soon after.

The fruits of the whole enterprise were published in the form of a multi-volume catalogue with plate parameters for a six constant model and the x and y coordinate

and magnitude for each star on that particular plate. The user could then from, the original measurements to determine the stars' equatorial coordinates and possibly further improve upon if then needed. The catalogue has a limiting magnitude between 12–13th magnitude, although with today's measuring devices this magnitude limit could probably be extended.

The United States Naval Observatory is currently entering all the coordinates and attempting to re-reduce the entire catalogue with a block adjustment. The author is in debt to the US Naval Observatory for providing the AC of the Orion region from their database. The reference catalogue used is the Astrographic Catalogue of Reference Stars (ACRS), also provided by the US Naval Observatory; who produced this catalogue solely for the purpose of re-reducing the whole of the Astrographic Catalogue. The density of the ACRS is an average of 8 stars per square degree, so nominally there will be 32 on each Astrographic plate and 8 on each McCormick plate (because of the bright star density in Orion this density is actually higher for this study).

Figure 8 shows the pattern of the San Fernando and Algiers photographic plates. The three belt stars of Orion are shown for orientation. Note the strong overlapping corner in center pattern. This means that a star's image can be on up to 5 separate plates.

The San Fernando region

We used 19 plates from the San Fernando region. Table 1 summarizes the original AC data. Listed are a running plate number, astrographic catalogue plate number, right ascension in hours, declination in degrees, epoch of Observation and the six parameters given in the AC.

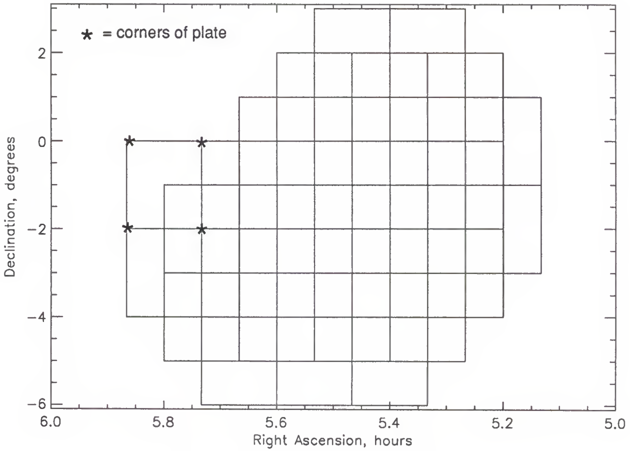


Figure 8: First Epoch Plate Orientation

By examining the epochs of the plates one can see that the majority of them were taken around 1893, and then there is another cluster of observations in 1909. The latter plates only cover stars above 0° declination; this will be drawn upon in the final summary. For now, we shall just note that the San Fernando region was made up of material clustered around two epochs, 1893 and 1909.

The first and fifth parameters, *a* and *e*, are quite large and positive; this implies that the focal length of the San Fernando astrograph was approximately 37mm smaller than the standard astrographic focal length of 3437mm.

Table 1: San-Fernando Astrographic Plates Data

#	AC#	α_oh	δ_o°	Epoch	Six constant model published with the AC					
1	2286	5.33	2.0	1910.0	37.75	3.02	0.144	-3.20	37.29	0.105
2	2287	5.47	2.0	1910.0	37.62	9.69	0.028	-9.94	37.17	0.068
3	1954	5.27	1.0	1909.0	37.22	-12.5	-.552	12.33	36.49	-.036
4	1956	5.40	1.0	1909.0	37.64	0.80	-.445	-0.87	36.88	-.124
5	1957	5.53	1.0	1909.0	37.20	9.13	0.022	-9.24	36.44	-.018
6	0621	5.20	0.0	1894.0	37.45	-8.72	0.454	8.75	36.89	-1.35
7	0135	5.33	0.0	1892.1	38.79	-1.70	-.948	1.70	38.27	0.179
8	0630	5.47	0.0	1894.0	36.80	6.77	0.242	-6.77	36.28	-.308
9	0136	5.60	0.0	1892.1	37.66	-13.6	-.319	13.62	37.11	-.409
10	0411	5.27	-1.0	1893.1	37.36	-42.6	1.023	42.80	36.77	-.515
11	0412	5.40	-1.0	1893.1	37.33	4.38	1.455	-4.31	36.77	-.475
12	0080	5.53	-1.0	1892.0	37.19	-19.5	-.139	19.56	36.60	0.099
13	0413	5.67	-1.0	1893.1	37.27	-10.4	0.202	10.52	36.68	-.263
14	0403	5.80	-1.0	1893.1	36.58	-40.7	0.272	40.59	35.99	-.589
15	0424	5.20	-2.0	1893.1	37.13	-18.2	0.418	18.10	36.54	-.483
16	0400	5.33	-2.0	1893.0	36.69	7.36	-.297	-7.36	36.07	1.530
17	1108	5.47	-2.0	1896.0	37.43	2.36	0.093	-2.33	36.84	-.897
18	0161	5.60	-2.0	1892.1	37.84	-7.16	-.289	7.19	37.21	-.442
19	0163	5.73	-2.0	1892.1	37.01	-6.32	0.297	6.18	36.38	-.196

The second and forth parameters, b and d, vary quite widely and are usually the opposite of each other. These represent the rotation terms and this variation is expected if one also takes into account that the third and sixth parameters remain quite small. The San Fernando observers and measurers were very careful to ensure the plate was well centered and reasonably careful to ensure consistent alignment of the plates with respect to the sky.

The Algiers region

From the Algiers observatory we used 12 plates. Table 2 summarizes the original AC data listed in the same format as table 1.

Table 2: Algiers Astrographic Plates Data

#	AC#	α_0h	δ_0°	Epoch	Six constant model published with the AC					
20	2450	5.27	-3.0	1896.1	-1.00	2.68	0.608	-2.75	-1.65	0.128
21	4223	5.40	-3.0	1909.1	-1.10	0.69	0.062	-1.13	-1.68	-.212
22	4224	5.53	-3.0	1909.1	-1.03	0.89	0.881	-1.31	-1.89	-.107
23	2453	5.67	-3.0	1896.2	-1.13	0.10	0.033	-0.38	-1.96	0.131
24	1498	5.80	-3.0	1894.0	-1.51	-1.24	0.183	1.34	-2.06	-.145
25	1697	5.33	-4.0	1894.2	-1.24	-1.65	0.141	1.48	-1.55	0.004
26	2451	5.47	-4.0	1896.2	-0.41	-1.06	-.535	0.72	-1.86	2.729
27	1716	5.60	-4.0	1894.2	-1.00	-0.96	0.079	0.10	-1.82	-.036
28	1728	5.73	-4.0	1894.2	-0.83	-2.71	-.138	2.30	-0.96	-.091
29	2429	5.40	-5.0	1896.1	-0.62	-0.31	2.604	-0.07	-2.13	-1.38
30	1496	5.53	-5.0	1894.0	-1.48	1.00	0.216	-1.03	-1.99	0.298
31	1512	5.67	-5.0	1894.0	-1.17	-2.20	0.268	1.82	-1.96	-.167

Once again the observations are clustered around two epochs, 1895 and 1909. For this study we must pick one epoch as a mean epoch of observation and ignore the fact that the plates were actually taken at separate epochs. This is not strictly necessary; because we have three epochs of observation, it would be possible to include a proper motion term with each observation. This would however vastly increase the computational work required and the number of free parameters per image. Considering that the literature cites the nominal measuring error of the AC at four microns, the minimum proper motion to be measurable above the measuring error would be 0.02 arcseconds per year. There are very few stars with this high a proper motion, so we

will use the simple approach of assuming the observations to be made at the middle epoch, which for these data is approximately 1900.

The first and fifth parameters, *a* and *e*, are small; this implies that the focal length of the Algiers astrograph was close to the standard astrographic focal length of 3437mm.

The second and forth parameters, *b* and *d*, do not vary much and also the third and sixth parameters are small. The Algiers observers and measurers were very careful in both centering and orienting the photographic plates during observing and measuring.

The Second Epoch

For the second epoch we used photographic plates exposed by Heinrich Eichhorn at the 26–inch refractor of the Leander Mc Cormick Observatory. This is an exceptional refractor with a long history in parallax determinations and other astrometric research. With a plate scale of 22″/mm it has a much larger plate scale than the standard astrograph giving more precise information per unit area.

During these observations Eichhorn exposed each region four times, rotating the plate once after the second exposure. This was for a number of reasons: more images of each star would reduce the centering error that is inherent in any measuring process; by rotating the plate he nullified the effects of emulsion slip, and any systematic observing errors that depended on the orientation; and finally he ensured that if any particular exposure was subject to a large guiding error there would be other exposures of that region. Figure 9 illustrates the orientation of the 67 regions he choose to observe.

Again note the overlapping pattern: each star will appear in a maximum of four regions, and because of the repeated exposures, each star has the potential to be measured up to 16 times. This gives a strong tie for the overlapping method to exploit.

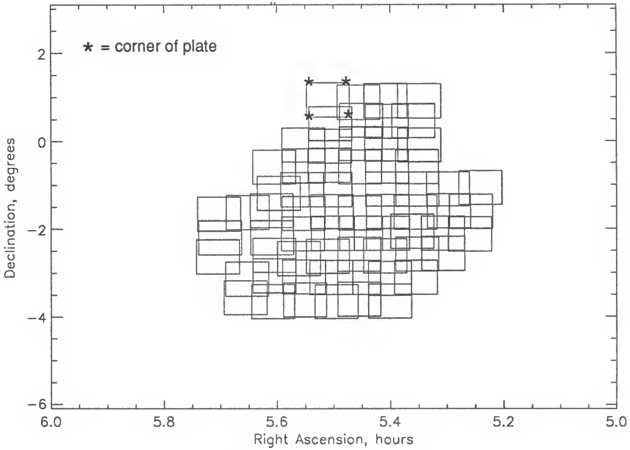


Figure 9: Second Epoch Plate Orientation

Table 3 summarizes the region data. Listed are running plate number; McCormick Observatory plate number; right ascension in hours; declination in degrees; date; eastern standard time; hour angle; seeing (1=high seeing, 5=low seeing); transmittance or cloud cover (1=very covered, 5=clear); and the temperature of the lens at the end of the exposure.

Table 3: Second Epoch McCormick Observatory Plates Data

#	MC#	α_oh	δ_o°	Epoch	EST	HA	S	T	Temperature
1	66807	5.66	-3.6	1955.7	09:13	0:40E	4	4	36
2	66810	5.61	-3.7	1955.7	09:39	0:11E	4	4	36
3	66812	5.55	-3.6	1955.7	10:01	0:17W	4	4	36
4	66814	5.55	-3.1	1955.7	10:22	0:36W	4	4	36
5	66816	5.61	-3.1	1955.7	10:45	0:54W	4	4	36
6	66817	5.65	-3.1	1955.7	11:00	1:06W	4	4	36
7	66833	5.40	-3.6	1956.1	08:28	0:56E	5	4	35
8	66835	5.46	-3.6	1956.1	08:48	0:39E	5	4	35
9	66837	5.50	-3.7	1956.1	09:19	0:10E	5	3	35
10	66839	5.50	-3.1	1956.1	09:51	0:22W	5	3	35
11	66841	5.46	-3.1	1956.1	10:23	0:56W	5	4	35
12	66843	5.71	-3.1	1956.1	10:51	1:09W	5	4	35
13	66853	5.40	-3.1	1956.1	08:27	0:51E	5	4	36
14	66855	5.36	-3.1	1956.1	08:47	0:28E	5	4	36
15	66857	5.35	-2.6	1956.1	09:10	0:05E	5	4	36
16	66863	5.51	-2.6	1956.1	10:19	0:54W	5	4	36
17	66865	5.71	-2.6	1956.1	10:48	1:12W	5	4	36
18	66884	5.26	-2.1	1956.1	07:53	0:56E	4	4	38
19	66886	5.26	-1.6	1956.1	08:01	0:48E	4	4	38
20	66895	5.41	-2.6	1956.1	08:16	0:15E	4	5	44
21	66897	5.46	-2.6	1956.1	08:34	0:00E	4	5	44
22	66899	5.56	-2.7	1956.1	08:55	0:16W	4	5	44
23	66901	5.61	-2.6	1956.1	09:11	0:28W	4	5	44
24	66911	5.31	-2.6	1956.1	07:40	0:40E	5	4	48
25	66913	5.31	-2.1	1956.1	07:58	0:22E	5	4	48
26	66915	5.36	-2.1	1956.1	08:29	0:05W	4	4	48
27	66917	5.40	-2.1	1956.1	08:45	0:18W	4	3	48
28	66919	5.30	-1.6	1956.1	07:05	1:11E	4	4	50
29	66921	5.36	-1.6	1956.1	07:25	0:54E	4	4	50
30	66923	5.40	-1.6	1956.1	07:46	0:36E	4	4	50
31	66925	5.45	-1.6	1956.1	08:07	0:18E	4	4	50
32	66927	5.50	-1.6	1956.1	08:29	0:01W	3	4	50
33	66929	5.55	-1.6	1956.1	08:48	0:17W	3	4	50
34	66932	5.55	-2.1	1956.1	09:12	0:41W	2	3	50
35	66933	5.61	-2.2	1956.1	09:34	0:59W	2	3	50
36	66942	5.24	-1.1	1956.1	06:56	1:06E	5	5	45
37	66944	5.30	-1.1	1956.1	07:13	0:51E	5	5	45
38	66946	5.35	-1.1	1956.1	07:35	0:32E	5	5	45
39	66948	5.41	-1.1	1956.1	07:53	0:17E	5	5	45
40	66957	5.45	-2.1	1956.1	07:26	0:43E	4	5	42

#	MC#	α_oh	δ_o°	Epoch	EST	HA	S	T	Temperature
41	66959	5.50	-2.1	1956.1	07:44	0:28E	4	5	42
42	66963	5.35	-0.1	1956.1	06:51	0:52E	3	5	52
43	66965	5.41	-0.1	1956.1	07:08	0:38E	5	5	52
44	66967	5.45	-0.1	1956.1	07:28	0:21E	5	5	52
45	66969	5.45	-0.6	1956.1	07:48	0:01E	5	5	52
46	66971	5.51	-1.1	1956.1	08:06	0:17W	5	5	52
47	66976	5.56	-1.1	1956.1	08:51	0:56W	5	5	52
48	66978	5.61	-1.6	1956.1	09:15	1:16W	5	5	52
49	66979	5.36	0.5	1956.1	06:48	0:47E	3	4	44
50	66981	5.41	0.5	1956.1	07:05	0:33E	3	4	44
51	66983	5.45	0.5	1956.1	07:25	0:16E	3	4	44
52	66985	5.51	0.4	1956.1	07:43	0:01E	3	4	44
53	66988	5.55	-0.1	1956.1	08:14	0:27W	3	3	44
54	66990	5.60	-1.2	1956.1	08:36	0:45W	3	3	44
55	66991	5.51	-0.1	1956.1	06:47	0:46E	4	4	42
56	66994	5.51	-0.6	1956.1	07:04	0:29E	4	4	39
57	66996	5.55	-0.6	1956.1	07:29	0:07E	4	4	39
58	66997	5.61	-0.6	1956.1	07:41	0:01W	4	4	39
59	66999	5.65	-1.6	1956.1	08:05	0:08W	4	4	39
60	67001	5.70	-2.2	1956.1	08:32	0:46W	5	4	37
61	67003	5.70	-1.7	1956.1					
62	67005	5.35	-0.6	1956.1					
63	67007	5.41	-0.6	1956.1					
64	67008	5.35	0.9	1956.1					
65	67010	5.41	0.9	1956.1					
66	67012	5.46	0.9	1956.1					
67	67015	5.51	0.9	1956.1					

Figure 10 is a picture of the refractor. The photographic plate holder is shown quite clearly at the base along with the offset guiding mechanism and exposure timer. At the top of the telescope one can see the housing for both the lens cover and the objective grating used in the second and third epochs.

The Third Epoch

For the third epoch we used plates exposed by Richard Smart, again on the 26–inch refractor of the Leander McCormick Observatory. For this epoch we recorded each

Figure 10: The Leander McCormick Refractor

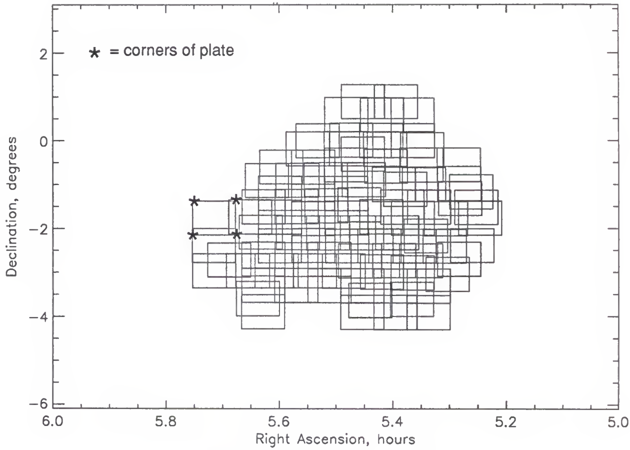


Figure 11: Third Epoch Plate Orientation

chosen region only an average of 3 times on a single plate as opposed to 4. However we covered 97 regions, allowing approximately the same coverage but with greater overlap. Figure 11 illustrates the orientation of the regions chosen.

Both the second and third epoch were observed using a large, 2.5 magnitude objective grating. This meant that most bright (i.e., brighter than 10th magnitude) stars formed up to 5 measurable stellar images. Later on we will discuss how these images were used to control the possible magnitude effects. Table 4 summarizes the region data listed in the same format as the second epoch data.

Table 4: Third Epoch McCormick Observatory Plates Data

#	MC#	α_oh	δ_o°	Epoch	EST	HA	S	T	Temperature
1	141225	5.28	-1.3	1991.9	12:45	0:25W	2	2	56
2	141227	5.26	-1.2	1991.9	13:14	0:10W	3	2	57
3	141229	5.38	-1.1	1991.9	14:04	0:25E	2	2	57
4	141231	5.65	-1.7	1991.9	14:36	0:50E	2	2	57
5	141247	5.25	-1.5	1991.9	12:31	0:50E	3	5	55
6	141249	5.38	-1.5	1991.9	13:00	0:30E	3	4	55
7	141251	5.46	-1.5	1991.9	13:38	0:05W	3	4	55
8	141253	5.52	-1.5	1991.9	14:05	0:30W	3	3	55
9	141263	5.35	-0.2	1991.9	12:34	0:40W	3	5	33
10	141265	5.41	-0.2	1991.9	13:03	0:20W	3	5	33
11	141267	5.48	-0.2	1991.9	13:34	0:08E	4	5	30
12	141269	5.55	-0.2	1991.9	13:56	0:36E	4	5	31
13	141279	5.28	-0.6	1991.9	12:51	0:20E	4	5	33
14	141281	5.35	-0.6	1991.9	13:18	0:04W	4	5	33
15	141283	5.41	-0.6	1991.9	13:42	0:23W	4	5	33
16	141285	5.48	-0.6	1991.9	14:06	0:45W	4	5	32
17	141341	5.36	0.0	1991.9	11:59	0:06E	3	5	42
18	141342	5.42	0.0	1991.9	12:17	0:10W	3	5	42
19	141343	5.47	0.0	1991.9	12:32	0:21W	3	5	42
20	141344	5.53	0.0	1991.9	13:00	0:45W	2	5	41
21	141345	5.25	-1.8	1991.9	11:10	0:44E	2	4	37
22	141346	5.31	-1.8	1991.9	11:37	0:20E	2	4	37
23	141347	5.36	-1.8	1991.9	11:57	0:03E	2	4	37
24	141348	5.42	-1.7	1991.9	12:13	0:10W	2	4	36
25	141349	5.48	-1.8	1991.9	12:36	0:28W	2	4	36
26	141350	5.54	-1.8	1991.9	12:46	0:36W	2	4	36
27	141351	5.60	-1.7	1991.9	13:04	0:50W	2	4	36
28	141352	5.72	-1.8	1991.9	13:20	0:58W	2	4	36
29	141353	5.34	-2.2	1991.9	11:25	0:30E	4	4	32
30	141354	5.39	-2.1	1991.9	11:42	0:15E	4	4	32
31	141355	5.45	-2.1	1991.9	11:59	0:03E	4	4	32
32	141356	5.51	-2.1	1991.9	12:24	0:16W	4	4	32
33	141357	5.57	-2.1	1991.9	12:47	0:34W	4	4	32
34	141358	5.63	-2.1	1991.9	12:57	0:46W	3	2	32
35	141368	5.26	-2.4	1992.0	11:02	0:40E	2	4	30
36	141369	5.31	-2.4	1992.0	11:29	0:16E	2	4	30
37	141370	5.36	-2.4	1992.0	11:55	0:06W	2	4	30
38	141371	5.42	-2.4	1992.0	12:19	0:28W	2	3	29
39	141372	5.48	-2.4	1992.0	12:36	0:40W	2	3	29
40	141373	5.54	-2.4	1992.0	12:57	0:58W	2	3	29

#	MC#	α_0h	δ_0°	Epoch	EST	HA	S	T	Temperature
41	141414	5.28	-2.7	1992.0	09:17	0:37E	3	3	32
42	141415	5.35	-2.7	1992.0	09:34	0:23E	3	3	32
43	141416	5.40	-2.7	1992.0	10:20	0:20W	2	3	31
44	141417	5.46	-2.7	1992.0	10:40	0:36W	2	3	31
45	141418	5.69	-2.7	1992.0	11:02	0:44w	2	3	31
46	141419	5.30	-3.0	1992.0	08:55	0:56E	4	3	25
47	141420	5.37	-3.0	1992.0	09:20	0:34E	4	3	25
48	141421	5.42	-3.0	1992.0	09:39	0:20E	4	3	25
49	141422	5.49	-3.0	1992.0	09:59	0:03E	4	3	25
50	141423	5.54	-3.0	1992.0	10:17	0:11W	3	3	22
51	141424	5.61	-3.0	1992.0	10:34	0:25W	3	3	22
52	141425	5.66	-3.0	1992.0	10:49	0:37W	3	3	22
53	141426	5.72	-3.0	1992.0	11:10	0:55W	3	3	22
54	141429	5.35	-1.0	1992.0	08:54	0:47E	3	3	22
55	141430	5.40	-1.0	1992.0	09:09	0:35E	3	3	22
56	141431	5.28	-1.0	1992.0	09:30	0:09E	4	3	28
57	141432	5.45	-0.9	1992.0	09:48	0:00E	4	3	28
58	141433	5.52	-0.9	1992.0	10:09	0:17W	4	3	28
59	141434	5.57	-0.9	1992.0	10:26	0:31W	4	3	28
60	141435	5.62	-0.9	1992.0	10:35	0:36W	4	3	28
61	141444	5.34	-1.5	1992.1	08:21	0:53E	3	4	34
62	141445	5.36	0.6	1992.1	08:38	0:37E	3	4	34
63	141446	5.42	0.6	1992.1	08:57	0:22E	3	4	34
64	141447	5.48	0.6	1992.1	09:21	0:02W	3	4	32
65	141448	5.45	-0.3	1992.1	09:36	0:16W	3	4	32
66	141449	5.51	-2.7	1992.1	10:00	0:37W	3	4	32
67	141450	5.57	-2.7	1992.1	10:15	0:47W	3	4	32
68	141455	5.34	-3.3	1992.1	08:30	0:36E	3	4	42
69	141456	5.40	-3.3	1992.1	08:46	0:23E	3	4	42
70	141457	5.45	-3.3	1992.1	09:03	0:10E	3	4	41
71	141458	5.51	-3.3	1992.1	09:24	0:09w	3	4	41
72	141459	5.57	-3.3	1992.1	09:38	0:18W	3	4	41
73	141460	5.63	-3.3	1992.1	09:55	0:32W	3	4	41
74	141461	5.60	-2.4	1992.1	10:15	0:54W	3	4	41
75	141471	5.48	-1.2	1992.1	09:20	0:09W	3	3	45
76	141472	5.54	-1.2	1992.1	09:34	0:20W	3	3	45
77	141473	5.60	-1.2	1992.1	09:45	0:27W	3	3	45
78	141474	5.57	-1.5	1992.1	10:00	0:45W	3	3	45
79	141475	5.63	-1.5	1992.1	10:14	0:55W	3	3	45
80	141490	5.39	0.9	1992.1	07:33	0:06E	4	4	44

#	MC#	α_oh	δ_o°	Epoch	EST	HA	S	T	Temperature
81	141491	5.45	0.9	1992.1	07:48	0:06W	4	4	44
82	141492	5.54	-0.6	1992.1	08:07	0:20W	4	4	44
83	141493	5.60	-0.6	1992.1	08:26	0:35W	4	4	44
84	141494	5.63	-2.7	1992.1	08:38	0:46W	4	4	44
85	141511	5.34	-3.9	1992.2	07:12	0:44w	4	4	62
86	141512	5.40	-3.9	1992.2	07:26	0:55w	4	4	62
87	141513	5.45	-3.9	1992.2	07:35	0:59w	4	4	62
88	141514	5.63	-3.9	1992.2	07:48	1:02W	4	4	62
89	141515	5.72	-2.4	1992.2	08:00	1:07E	4	4	62
90	141522	5.37	-3.6	1992.2	07:17	0:50W	2	4	34
91	141523	5.44	-3.6	1992.2	07:25	1:00W	2	4	34
92	141524	5.64	-3.6	1992.2	07:37	0:55W	2	4	34
93	141525	5.68	-2.1	1992.2	07:42	1:05W	2	4	34

Measuring the Plates

We measured the plates on the University of Minnesota Automated Plate Scanner (APS) at Minneapolis, which was originally designed as a proper motion measuring engine for Luyten's automated proper motion surveys. It has been refitted with modern electronics and is currently being used for digitizing the Palomar Sky Surveys.

The machine is shown in two pictures in figures 12 and 13 (figure 12 shows the overall machine.) It can measure two plates at any one time on the two measuring levels. For measuring the second and third epoch plates we used just the upper level. Figure 13 shows the laser beam that measures the plate density as the whole housing scans across the plate. Inside this housing is an eight sided prism that rotates, with the speed of the rotation setting the scanning speed. It scans out 12mm strips with 12 micron diameter spots in y and simultaneously the strip is moved in x. Thus knowing the speed of rotation, the motion in x, and the density at any point a digitized map of the plate can be made.

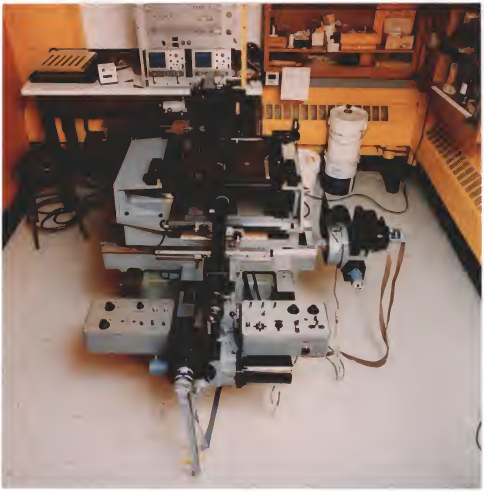


Figure 12: The Automated Plate Scanner(Pennington *et al.*, 1993)

Figure 13: The APS Scanning Laser (*Pennington et al.*, 1993)

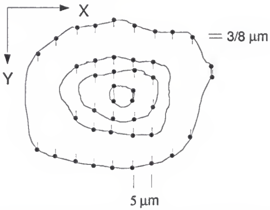


Figure 14: APS Isodensitometric scan (Pennington *et al.*, 1993)

In addition to differing speeds (and as such differing resolutions) there are three different kinds of scanning modes for the APS: isodensitometric, densitometric and a hybrid of these two modes. For measuring these plates we used the isodensitometric mode. In this mode the diffraction pattern of the laser beam as it scans is focussed on a photodetector. Every time the image attains either a preset or a dynamically set threshold density, the position is read to a precision of 0.366 microns. Therefore each stellar image will have two points for each scan across it, the ingress and egress of the image. Figure 14 shows a typical stellar image scanned in isodensitometric mode with four threshold levels.

An ellipse is fitted to this isodensitometric outline. This best fit ellipse then defines the image and its center defines the position of the image. The final output consists of six parameters for each image: x and y coordinates of the ellipse center, diameter, ellipticity, theta angle (angle of the best fit ellipse's semimajor axis to the x axis), and a value that measured the goodness of fit of the ellipse. For a typical plate there may be 5000 images of which only 1000 were real stellar images or grating images.

This machine is extremely fast and allowed us to measure all the plates twice in

the span of two weeks. For the first scan we only used one level threshold; for the second scan we used two thresholds. So for each plate we have three estimates of the images on that plate.

Each adjacent stripe has a 1mm overlap region. Images in this overlap region that have matching parameters are assumed to be the same image and then used to tie stripes together. This is because the machine would tend to drift by perhaps 2–3 microns between stripes and the use of an overlap allowed a continual mapping of the measuring frame to a fixed frame.

Figure 15 shows the predicted stellar positions for the regions 141368, 141369 and 141370. Each region was exposed three times on the same emulsion. One emulsion was used due to their high cost, it also made the scanning easier as we scanned the whole plate in one 'sweep'. Each plate is filed under the McCormick number of the first exposure.

Each star with a cross is a reference star; they are brighter on average than the field stars. The grating images of the brighter stars ($m<10$) are used to ensure that any magnitude effects are equally modelled for the whole of the stellar magnitude range. With ideal magnitude-independent imaging, the primary stellar image will be at the average of the first and second order images. Coma and magnitude effects will cause deviations from an exact agreement of the average of the grating images and the primary images. By examining this deviation we can very accurately model the magnitude dependent terms. (There is no better way to do it.)

There are different ways of examining this deviation. In this analysis for every measured objective grating pair we assigned a fictitious star to their average. Figure 16

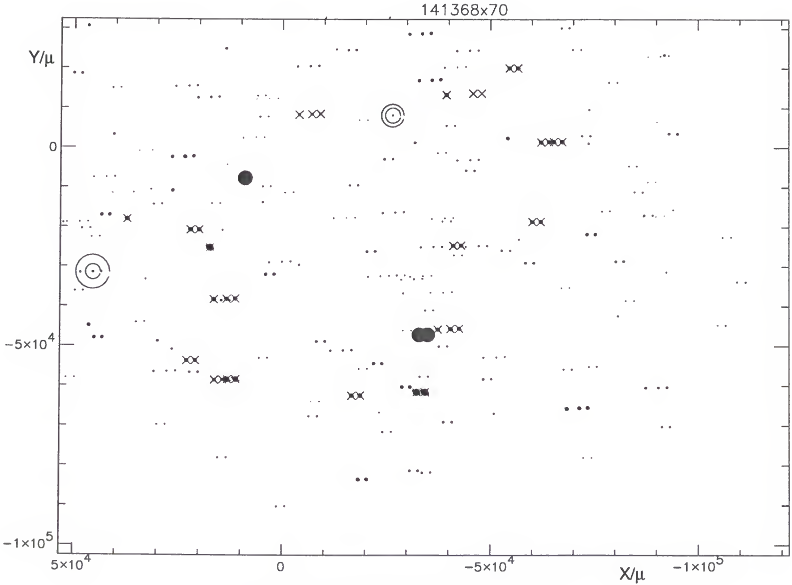


Figure 16: Predicted Positions for Plate 141368x70

shows an example of a difference that might be expected. The **X** marks the average of the two first order grating images. At this point we 'create' a fictitious star that has all the same parameters as the primary star except its magnitude is reduced by the grating constant. This is then entered into the overlap as an extra observation and the information contained in its position and magnitude will be enforced by the overlap least-squares reduction.

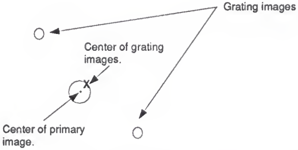


Figure 16: The difference between a Grating Image Average and the Primary Image Center

This method may increase the variance of the solution, but we feel the possible increase in variance due to extra observations will be outweighed by the potentially stronger overlap and the ability to solve for all the parameters simultaneously.

Figure 17 shows the scan of plate 141368. This shows one drawback of using a diffraction grating. The large ellipses are blended diffraction images that the software has been unable to differentiate. These images were not used in the stellar positions and this results in a minor loss of data. As we have 3–4 exposures of each region, and because a particular star may appear in perhaps 4 regions then the loss of 1 or 2 of its measurements will not greatly affect the final positions.

To find the location of the bright stars that are differentially subject to image blending in the second set of scans (the reverse scans), we used two thresholds. Figure

18 shows the same plate scanned again in the upper threshold. As the figure shows we have lost the dimmer stars and grating images, but the brighter stars have been resolved. This was treated as a separate exposure of the same region, again allowing a strong overlap and actually allowing an independent check of the plate parameters; they should be the same for the second scan.

Summary of Observational Material and Measured Images

To summarize, the observational material basically consists of photographic plates exposed at three epochs. Below are the observation statistics for all three epochs:

First Epoch Plate Information	
Observatory and epoch	San Fernando and Algiers, 1900
Number of Regions	31
Number of Exposures	31
Plate scale of telescope	60″/mm
Average Positional precision	4μm
Second Epoch Plate Information	
Observatory and epoch	McCormick Observatory, 1956
Number of Regions	67
Number of Exposures	261
Plate scale of telescope	22″/mm
Average Positional precision	2μm
Third Epoch Plate Information	
Observatory and epoch	McCormick Observatory, 1992
Number of Regions	93
Number of Exposures	293
Plate scale of telescope	22″/mm
Average Positional precision	2μm

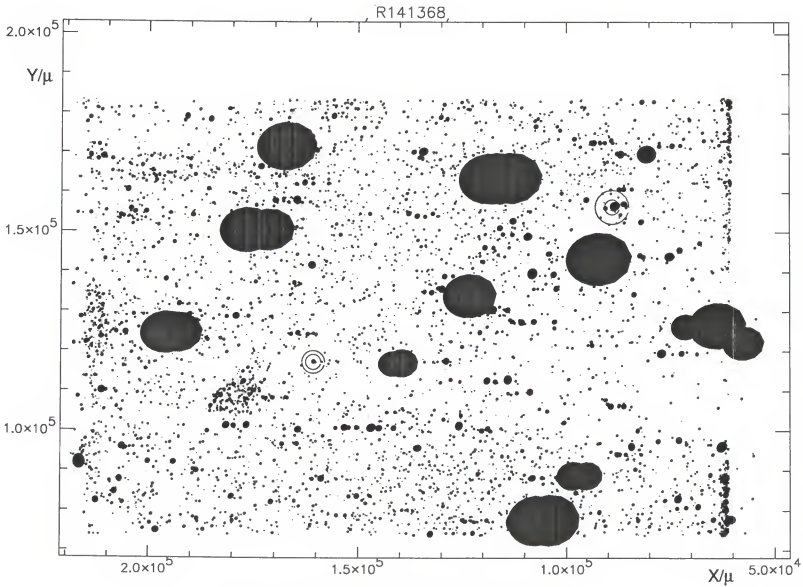


Figure 17: Scan of Plate 141368x70 Low Threshold

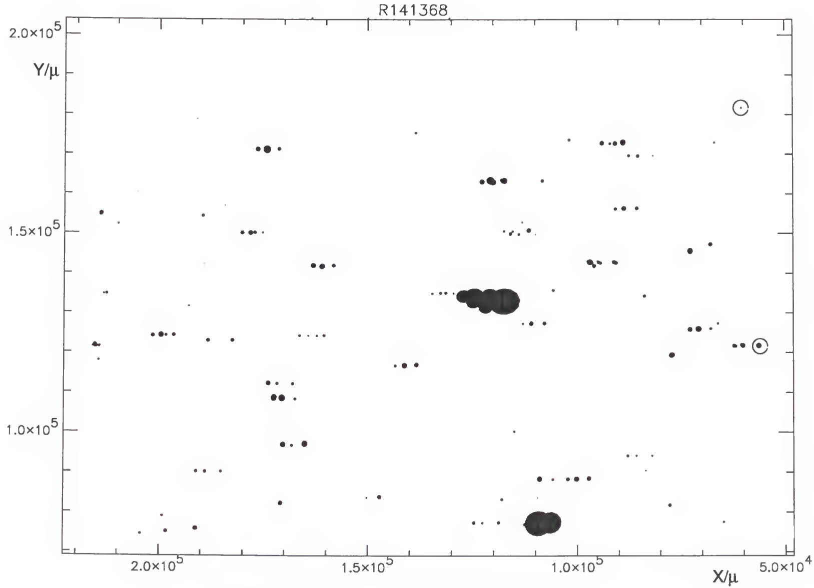


Figure 18: Scan of Plate 141368x70 High Threshold

CHAPTER 5
FINDING A MODEL

In the discussion on Finding Stellar Coordinates we used as an example the six-constant model. However, as we also discussed, to account accurately for the complex relationship between the observations (x,y) and the standard coordinates (ξ,η) a greater number of terms is needed. The model will invariably not account for all contributing effects. There comes a point when model improvement will require more effort than the potential improvement in accuracy warrants. Also, applying a complicated model known to be correct but which requires the estimation of a relatively large number of parameters may depress the ultimate accuracy due to generating a large parameter variance, (cf. Eichhorn and Williams, 1963).

For the choice of a model there is no unambiguous technique that will give black and white indications of which parameters to include. The procedure is more of an art than a mathematical reduction. In addition to the numerical tests of significance an understanding of the physical situation aids the analysis significantly.

The discussion will be split into sections: previous studies, position residuals analysis, parameter variation analysis, and solution variances. There will also be a separate discussion of single and overlapping plate solutions, this is important because the noise in the single plate solution may hide some effects and the stronger tie in the overlap may also hide some different effects. These sections are chosen to simplify the discussion. The final choice was based on an examination all of these indicators simultaneously, while trying out different models.

Previous Work

The original publications of the Astrographic Catalogue included the basic six-constant models. This was because the accuracy required only warranted a six-constant model and the number of reference stars per-square degree never really allowed a larger model. Using the ACRS there is an average of 32 reference stars per plate. This will give us only a parameter-to-observation ratio of two in a 16–constant single plate solution. This ratio is much higher in the overlap where there are on average 200 images per plate, many of which will act as observations. Therefore for reasonable size models by using an overlap solution we are no longer limited to a small model and we can examine what other possible terms may be included.

Previous work on zones of the AC have used simple six-constant models to ten-constant models (Eichhorn and Gatewood, 1967) all the way up to a combination of the six-constant model with twenty constrained constants (Gunther and Kox, 1971). Various investigations have found that AC material contains evidence for magnitude terms, color terms, coma terms and, radial terms.

In this analysis we deal with the Algiers and the San Fernando zone. During the measuring of the Algiers zone no magnitude term was found (Eichhorn and Clary, 1974). In the San Fernando zone the measurements showed no signs of radial distortion, aberration or refraction. (It is worth noting however that high discrepancies, up to 25μ, between rectangular coordinates of the same image were tolerated, and thus these effects could easily be lost in the noise.)

The astrometric properties of the McCormick Refractor were examined by Russell (1976). She examined data from 15 large astrometric telescopes for evidence of

model terms in magnitude, coma, magnitude squared, color, color magnification (color-coordinate product), tilt, refraction (position cross product), distortion and, magnitude-distortion product. The McCormick Telescope showed significant terms in the magnitude, coma, tilt, refraction, and the magnitude–distortion term. When we finalize our model choice we will refer to the previous work to see if they agree with previous investigations.

Positional Residuals

In this section we will analyze the output from six-constant solutions. The objective is to use a very basic model and see what extra terms the position residuals and errors are correlated with. We analyze 21 extra terms by graphing the residuals of our solution vs their value. The 21 extra terms analyzed were

$$\begin{gathered}(m-\langle m\rangle),(m-\langle m\rangle)^2,(m-\langle m\rangle)x,(m-\langle m\rangle)y,\\(m-\langle m\rangle)x^2,(m-\langle m\rangle)y^2,(m-\langle m\rangle)^2x,(m-\langle m\rangle)^2y,\\(m-\langle m\rangle)(x^2+y^2),(m-\langle m\rangle)x(x^2+y^2),\\(m-\langle m\rangle)y(x^2+y^2),(m-\langle m\rangle)(xy),xy,x^2,y^2,\\x^2y,xy^2,(x^2+y^2),x(x^2+y^2),y(x^2+y^2),(x^2+y^2)^2.\end{gathered}\tag{5.1}$$

In the analysis of each term we examined a host of various graphs. It would be counter productive to reproduce all of the graphs, examples are reproduced for each technique.

Single Plate Residuals

Consider the results from a single plate solution. For the reference stars the residuals can be found and are of the form

$$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} - \left(s \begin{pmatrix} \xi \\ \eta \end{pmatrix} + \Xi a \right) \tag{5.2}$$

where the x, y, ξ and η are the inputted values and the parameter matrix is the result of the reduction. These residuals can then be examined to see which other terms in the stellar parameters may be significant. In figures 19 and 20 we graph the residuals (Δx and Δy) of a six–constant single plate solutions vs the different extra terms listed above. The residuals are for 27 plates of the 1900 epoch material; all plates with over 25 reference stars were included.

The residual on the y axis in each graph is limited to three standard deviations, the dotted line represents two standard deviations. There is a large amount of scatter in the points. This is expected as ideally they would be randomly distributed reflecting the problems in the reference data and measuring process. There may also be some misidentifications in the sample as well which will add to the noise.

A 3rd order polynomial was fitted to the points, for the purpose of examining trends in the curve. Graphs that were similar, both in scale and trend, were noted. For example, if we found a trend in a term that was similar in both the x and y residual then we can assume the parameter for this term, should we include it, would be the same.

From figure 19 we can make the following conclusions: the residuals are correlated with both the m (magnitude) and m^2 quantities, the correlation is stronger and consistent in the Δy, and not as strong or consistent in the Δx. There is no strong evidence for

Figure 19: Six-Constant Single Plate Residuals

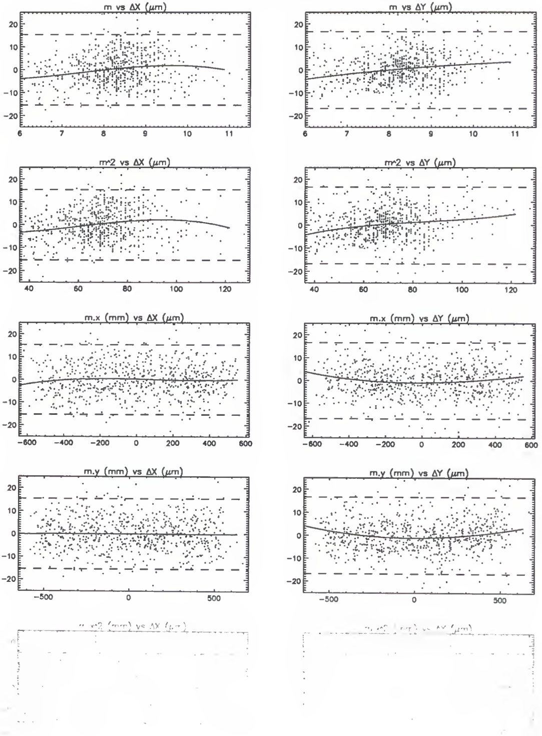
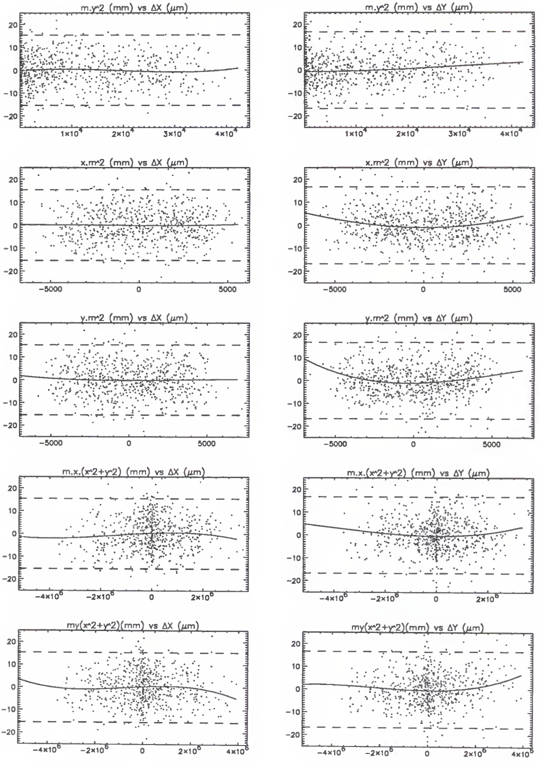


Figure 20: Six-Constant Single Plate Residuals



a correlation in the Δx vs mx, my or mx^2. In the Δy vs mx and my there is evidence for a correlation and only slight evidence in the Δy vs mx^2.

From figure 20 we can see that the Δx is not consistently correlated in any of the terms, though it may be in the $mx(x^2+y^2)$ term. The Δy shows a strong correlation in my^2, xm^2, and ym^2 and a weaker correlation in the $mx(x^2+y^2)$ and $my(x^2+y^2)$ terms.

These correlations imply that the plates suffered from a guiding error and some coma, mainly in the y coordinate. Based on these graphs, for the 1900 model, we should include terms in the x coordinate that fit m, m^2 and prehaps mx, and in the y coordinate terms that fit m, m^2, mx, my, xm^2, and ym^2. In the actual development we would not do this straight away. First we would examine the corresponding six-constant model overlap residuals (as in the next section), and then if there were still the same correlations we would fit a magnitude term and one or two of the other terms. Then we reexamine the remaining terms and add terms as needed.

Overlap Residuals

In the single plate method because each solution was only dependent on 20–30 reference stars, the solutions could be loose and therefore some of the trends could have been hidden in the noise. In the overlapping plate method the solution is much stronger because of the restriction that a star have the same position from plate to plate. Therefore, even though we are only using a six-plate model some effects, such as radial ones, will be accounted for by the overlap.

For example, the images of a given star at the centers of one plate must have the same position as an image of the same star that may be in the corner on another plate at any give epoch. Any radial effects will be canceled out when we find the mean

weighted position. When we find the residual of the position this effect is slightly nullified because we are returning again to single images rather than mean weighted average star positions.

Now we examine the residuals from a six-constant overlap as we did in the single plate example to see if there are any improvements or any other problems highlighted. Figures 21 and 22 show examples of residual graphs for a six-constant overlap of the 1900 material.

In figure 21 we again see the same positive correlation in the m and m^2 terms, though it is damped slightly. There does however seem to be no correlation the other terms except for the my term which shows a strong parabolic shape; this implies a very real trend.

In figure 22 we see some trends that we had thought were not consistent, like that in the Δx vs $mx(x^2+y^2)$ and $my(x^2+y^2)$ terms and we also see a new similar feature in the xm^2 term. If we decide to fit the $mx(x^2+y^2)$ and $my(x^2+y^2)$ for either Δx or Δy we can keep the same parameters for them as the correlation is identical. The only other correlation is in the Δy vs ym^2 term which repeats the correlation seen in the single plate analysis and should be modelled.

Summary

From this analysis we can highlight those terms we feel important for further analysis. After the more sophisticated model is run, another examination of the residuals may highlight more correlations and these will have to be included. An analysis of the parameter variation in the next section shows how we can highlight parameters that may be correlated but are not significant to the reduction and therefore can be removed.

Figure 21: Six-Constant Overlap Residuals

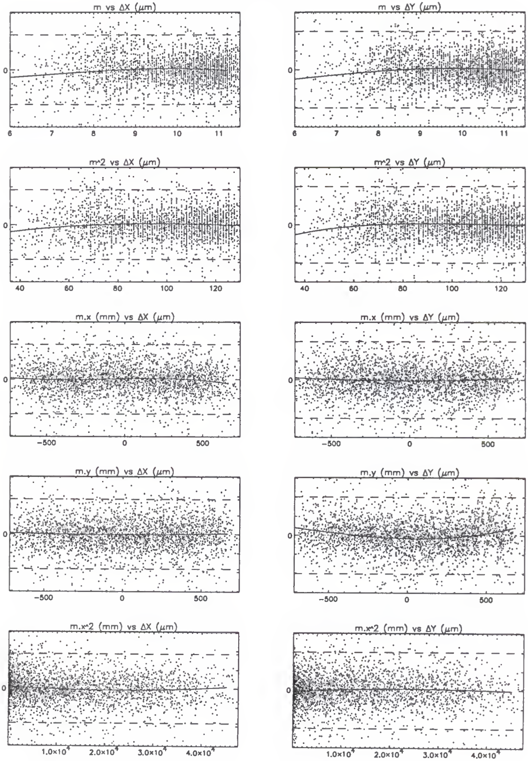
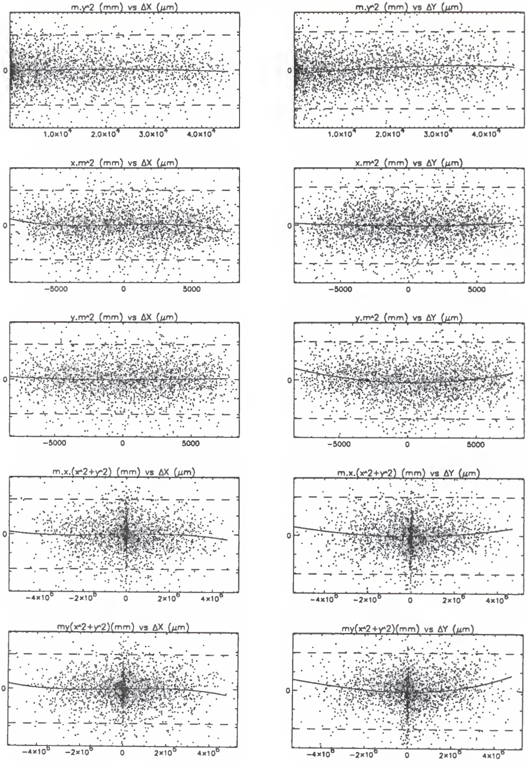


Figure 22: Six-Constant Overlap Residuals



From our residual analysis we found that all three epochs showed significant first order magnitude effects and so the minimum model would be the eight constant model:

$$\Xi = \begin{pmatrix} \xi & \eta & 1 & 0 & 0 & 0 & m-\langle m\rangle & 0 \\ 0 & 0 & 0 & \xi & \eta & 1 & 0 & m-\langle m\rangle \end{pmatrix}, \quad a = \begin{pmatrix} a \\ b \\ c \\ a' \\ b' \\ c' \\ e \\ f \end{pmatrix}. \tag{5.3}$$

Now for each of the epochs we found different effects and they are discussed separately.

Epoch 1900: An analysis of the residuals showed the terms were correlated mainly with the Δy residual. This Δy varied so strongly in one term the Δx residual is not modelled but the Δy is. A very strong correlation was also found in the first and second order magnitude terms that could possibly be modeled by only two parameters. There is a consistent and similar variation in the mx(x^2+y^2) and my(x^2+y^2) terms indicating they should only be modeled in y and that it may be the m(x^2+y^2) term that needs modelling. We found a similar but opposite (e.g. acting on the opposite residual) in the xy and mxy terms, signifying these can be modeled by only two parameters, similarly in the xy^2 and x(x^2+y^2) terms. Finally correlations were different in the Δy and Δx vs. (x^2+y^2), hence two parameters were needed for this one radial term.

After residual analysis we decided to try the eight constant model plus the following eight terms:

$$\begin{matrix} \xi\eta & (\xi^2+\eta^2)\xi\eta & (m-\langle m\rangle)^2 & 0 \\ (\xi^2+\eta^2)\xi\eta & \xi\eta & (m-\langle m\rangle)^2 & (m-\langle m\rangle)\eta \end{matrix}$$

$$(5.4)$$

$$\begin{matrix} \xi\eta^2 & \xi(\xi^2+\eta^2) & (\xi^2+\eta^2) & 0 \\ \xi(\xi^2+\eta^2) & \xi\eta^2 & 0 & (\xi^2+\eta^2). \end{matrix}$$

Epoch 1956: For the 1956 material there was very little evidence for magnitude and tilt terms. The coma and magnitude distortion terms seem to dominate, but even these cause very little variation. After the residual analysis we decided to examine the following terms:

$$\begin{array}{cccc} (m-\langle m\rangle)\xi(\xi^2+\eta^2) & 0 & (m-\langle m\rangle)\eta(\xi^2+\eta^2) & 0 \\ 0 & (m-\langle m\rangle)\xi(\xi^2+\eta^2) & 0 & (m-\langle m\rangle)\eta(\xi^2+\eta^2) \end{array}$$

$$\begin{array}{ccc} (m-\langle m\rangle)^2\xi & 0 & (m-\langle m\rangle)^2(\xi^2+\eta^2) \\ 0 & (m-\langle m\rangle)^2\eta & (m-\langle m\rangle)^2(\xi^2+\eta^2). \end{array} \tag{5.5}$$

Epoch 1992: Again dominated by coma and magnitude terms; in this case however the graphs are similar and we have modelled these terms with single parameters. There was some slight slope in the residual plots for the tilt terms and origin–distance squared terms but not very pronounced. After residual analysis we decided to examine the following terms:

$$\begin{array}{cc} (m-\langle m\rangle)\xi(\xi^2+\eta^2) & (m-\langle m\rangle)\eta(\xi^2+\eta^2) \\ (m-\langle m\rangle)\eta(\xi^2+\eta^2) & (m-\langle m\rangle)\xi(\xi^2+\eta^2) \end{array}$$

$$\begin{array}{ccc} (m-\langle m\rangle)^2\xi & 0 & (m-\langle m\rangle)^2(\xi^2+\eta^2) \\ 0 & (m-\langle m\rangle)^2\eta & (m-\langle m\rangle)^2(\xi^2+\eta^2). \end{array} \tag{5.6}$$

Note that even though the residuals may not imply there is a magnitude term we have included one. This is because the magnitude effect has a strong tie to the particular guiding in an exposure; all the plates were hand guided, and therefore the term is likely

to be randomized over the entire plate system. As such it may not show up in this kind of analysis; on the other hand if it is negligible then an analysis of the parameter variation will highlight this.

Two things to note about the models: the astrographic telescopes do not show particular coma or magnitude distortion terms, but do show some radial distortion and tilt terms. The 1956 and 1992 models agree quite well, as one would expect when using the same telescope. They also contains terms in magnitude distortion and coma found by Russell (1976), though there was no strong evidence for tilt terms as she found.

Parameter Variation

Now we have picked the terms we feel are correlated with the residuals and should therefore be modeled. We should now check to make sure all the terms are needed. It would be quite possible for a term to be highly correlated but still have a negligible effect. Removing extra terms is important for two considerations:

1. As one increases the number of parameters their variance will also increase. This will propagate an error directly into the calculated positions. Eichhorn and Williams showed that parameter variance can actually make a correct model give less accuracy than a smaller approximate model.
2. In the physical process of computation every calculation results in a small round-off error. If we consider a overlap of 30 plates with 12 parameters, then the matrix of normal equations has dimension (360×360). If we now include 2 more parameters the dimension increases to (420×420). A good rule of thumb is that the inversion of a matrix will require N^3 multiplications (where N is the order of the matrix). Therefore, continuing with our example the inclusion of 2 extra parameters

will require $(420^3 - 360^3) = 27{,}432{,}000$ extra calculations, nearly half as many multiplications as those needed for the 12 constant model. The loss of accuracy incurred by round off error in the increased computations may well outweigh the potential accuracy increase due to using a better model.

The first examination must be made with the single plate analysis, because including too many terms in the overlap will cause the inversion process to 'blow up' and the matrix will not be invertible. This examination quantifies the graphically based subjective choices made above. It will enable a simplifying of the potential models by highlighting parameters that have negligible effects (even highly correlated ones) and parameters that do not change from plate to plate.

The parameter's mean and standard deviation will indicate how well determined and significant the parameter is. The mean parameter and the largest ξ and η values will provide an upper bound on the term's contribution . For the AC the largest standard coordinate value is ξ,η=0.038 radians and for the McCormick data ξ=0.018 and η=0.013radians.

The errors of the individual parameters will also be important to examine. These will be given by the square root of the diagonal term of the covariance matrix. In a system of equations represented by $\mathbf{A}a = x_o$ the covariance matrix, **C**, is proportional to the inverse of the matrix **A**. Referring back to the single plate equation (3.23)

$$a = \left(\sum_{\nu=1}^{m} \Xi_\nu^{\mathrm{T}} \mathbf{J}_\nu \Xi_\nu \right)^{-1} \sum_{\nu=1}^{m} \Xi_\nu^{\mathrm{T}} \mathbf{J}_\nu d_\nu$$

here the matrix $\left(\sum\limits_{\nu=1}^{m} \Xi_\nu^{\mathrm{T}} \mathbf{J}_\nu \Xi_\nu \right)^{-1}$ is proportional to the covariance matrix.

Finally, terms should be examined to see if they are correlated. To do this we calculate the correlation parameter: $\frac{c_{\nu\mu}}{\sqrt{c_{\nu\nu}.c_{\mu\mu}}}$ where $c_{\nu\mu}$ represents the (μ,ν) term of the covariance matrix.

Table 5 summarizes all these quantities for the single plate solutions using the models outlined in 5.4, 5.5, and 5.6. As in the single plate solution we only use the reference stars, these parameters will not be very well determined, but their general variation will provide information for the removal or constraining of parameters as required. Listed are

#:	Parameter number, 7 and 8 represent first order magnitude terms, the other parameters are as given in equations 5.4, 5.5, and 5.6.
Mean:	Mean parameter value, if this is very large it means the parameter will be dominating the calculation of the whole parameter vector.
σ:	Standard deviation of the parameter value, if this is greater than the mean value then one must consider if the parameter can be ignored.
Range:	Range of the parameter, indicates if there are some aberrant values.
Mean ϵ:	Mean parameter error, this is the square root of the covariance matrix diagonal term averaged for all the plates.
σ_ϵ:	Standard deviation of the error.
Mean ϵ / σ:	Ratio of the mean error to the parameter standard deviation. If this is equal to one then the parameter is varying within its error.
Mean / Mean ϵ:	Ratio of the mean parameter value to the mean error. If this is equal to one then the parameter is within its error, zero.

σ_ϵ/σ: Ratio of error to parameter standard deviation. If this is one, 65% of the values are within their error the mean value.

Term Mean: The highest value of the model term if you use the mean value of the parameter.

Term Range: The range of that model term.

We can draw some general conclusions from these quantities, but final judgement should be carried out simultaneously with the graphical representations and with the residual analysis material. The terms to be concerned about are the large parameters. If we included these in the overlap, then they will dominate the covariance matrix and lead to a very unstable solution. If they are only varying within their error then they can be constrained and as such will not effect the covariance matrix as strongly. If they vary too wildly then it may be that, with our data, particular term cannot be realistically modelled.

The terms 9, 10, 11, 12 and 9, 10 respectively in the 1956 and 1992 material vary widely. These model the magnitude distortion terms: $(m - \langle m \rangle)\xi(\xi^2 + \eta^2)$ and $(m - \langle m \rangle)\eta(\xi^2 + \eta^2)$. These terms being cubic in the standard coordinates are very small, and their parameters have to be corresponding large, but this does cause problems for the inversion process. Since this term is a very system dependent term, e.g. dependent on the optics more than the observing / guiding, it may be physically realistic to constrain it to one parameter for all the plates.

Term 13 in the 1992 material also seems to be very discordant, it has a small value, a reasonably small error and error standard deviation; yet it has a large value standard deviation and very large range. This may have some discordant values that are

skewing the overall distribution. The graphical examination also provides information with which to answer some of these concerns.

Graphical Analysis of Parameters

Figures 23, 24, and 25 show the variation of the first eight extra parameters for a the same sample of plates from each epoch. This is a graphical representation of the parameters just discussed. The y axis represents the value of the parameter, and the x axis is a running plate number. The order of the plates corresponds to the date of exposure. On each parameter point, the length of the vertical line represents the formal error, e.g. the square root of the corresponding diagonal term on the covariance matrix. The dotted line represents a two standard deviation level for each parameter. The line through the middle is a best fit horizontal line.

As one can see the size, range and, scatter varies quite a lot. This 'scatter' is quantified by the ratio of the parameter standard deviation to its mean covariance in table 5. This ratio will indicate if a parameter is varying outside its formal error. If this is not the case then, providing the variation is due to random causes, we can assume the variation is within the noise and should either be constrained or dropped completely.

Table 5: Parameter Statistics

#	Mean	σ	Range	Mean ϵ	σ_ϵ
	==================		Epoch 1900	==================	
7	-0.00155	0.00219	0.0104	0.000787	0.000285
8	-0.00210	0.00173	0.00869	0.000837	0.000318
9	0.148	15.0	58.0	7.03	2.27
10	11.3	13.7	58.0	7.21	2.33
11	0.000276	0.000755	0.00368	0.000447	0.000335
12	0.0285	0.141	0.788	0.0615	0.0499
13	-28.2	1.21e+03	6.07e+03	576.	137.
14	-490.	1.27e+03	4.97e+03	572.	135.
15	4.58	10.7	42.1	5.11	1.18
16	-8.26	11.4	53.6	5.15	1.18
	==================		Epoch 1956	==================	
7	-7.17e-05	0.00512	0.0522	0.000778	0.000432
8	0.000473	0.00427	0.0395	0.000783	0.000386
9	1.07e+04	3.21e+04	3.47e+05	6.62e+03	6.28e+03
10	-562.	3.40e+04	3.59e+05	6.28e+03	6.45e+03
11	3.72e+03	5.49e+04	6.18e+05	1.10e+04	1.59e+04
12	2.24e+04	6.11e+04	6.51e+05	1.14e+04	1.62e+04
13	0.0307	0.710	6.47	0.146	0.102
14	-0.0540	1.11	9.84	0.220	0.205
15	-1.56	80.1	651.	15.5	15.9
	==================		Epoch 1992	==================	
7	-0.000745	0.0106	0.112	0.000666	0.000270
8	0.000319	0.00816	0.0797	0.000665	0.000271
9	1.44e+04	1.21e+05	2.01e+06	4.25e+03	4.50e+03
10	1.07e+04	1.22e+05	2.12e+06	3.98e+03	4.15e+03
11	-0.183	2.74	27.2	0.146	0.113
12	-0.202	3.35	33.5	0.213	0.195
13	21.2	302.	4.21e+03	13.2	12.1

Table 5: (Continued) Parameter Statistics

#	Mean ϵ / σ	Mean / Mean ϵ	σ_ϵ / σ	Term Mean	Term Range
	===================		Epoch 1900	===================	
7	0.358	1.97	0.130	-0.0186	0.125
8	0.485	2.51	0.184	-0.0252	0.104
9	0.468	0.0210	0.151	0.000214	0.0838
10	0.526	1.57	0.170	0.0163	0.0837
11	0.592	0.617	0.444	0.0397	0.529
12	0.437	0.463	0.355	0.0130	0.359
13	0.478	0.0489	0.113	-0.00309	0.666
14	0.450	0.857	0.106	-0.0538	0.545
15	0.478	0.897	0.111	0.0132	0.122
16	0.451	1.60	0.103	-0.0238	0.155
	===================		Epoch 1956	===================	
7	0.152	0.0922	0.0844	-0.000861	0.626
8	0.184	0.605	0.0906	0.00568	0.474
9	0.206	1.61	0.195	1.14	36.9
10	0.185	0.0895	0.190	-0.0598	38.3
11	0.201	0.338	0.289	0.286	47.5
12	0.186	1.96	0.265	1.72	50.1
13	0.206	0.210	0.143	0.0796	16.8
14	0.199	0.245	0.185	-0.101	18.4
15	0.194	0.101	0.199	-0.111	46.2
	===================		Epoch 1992	===================	
7	0.0631	1.12	0.0256	-0.00894	1.34
8	0.0814	0.480	0.0332	0.00382	0.957
9	0.0351	3.39	0.0371	1.54	214.
10	0.0326	2.68	0.0340	1.14	226.
11	0.0532	1.26	0.0412	-0.475	70.5
12	0.0634	0.949	0.0581	-0.378	62.7
13	0.0438	1.60	0.0401	1.51	299.

There are circumstances when that will not be the case. For example in the case of the straight magnitude parameters, e and f, these are primarily terms that result from bad guiding. If they are included (e.g. if they are significant) then, because they are very plate / observer dependent, constraining them would not make physical sense. The two terms may not however be significant, if they vary around a mean of zero, and their standard deviation is less than their formal error they can be dropped.

The graphs confirm that the parameters for magnitude distortion can defiantly be constrained for the 1992 epoch (parameters 9 and 10), and probably constrained for the 1956 epoch (parameters 9,10,11,12). More analysis of this nature must be carried out to make a definite decision.

An examination of this nature to find the model is very convoluted. This discussion has shown how we can use the residuals and parameters to indicate which model to use. The whole choice is, as we have said, not black and white. After iterating through tests on parameters and residuals simultaneously we will eventually see a pattern of terms that need to be included. At some point the decision will be made that the potential improvement in precision does not warrant further work on the model. There is one last reduction improvement we can make after this point to improve the solution, this involves analyzing the output variances.

Below are the models we have decided upon for this investigation, for all three epochs we included the basic eight constant model. For epoch 1900 we included as extra terms the following:

$$\begin{array}{lcc} (\xi^2+\eta^2)\xi\eta & (m-\langle m\rangle)^2 & 0 \\ \xi\eta & (m-\langle m\rangle)^2 & (m-\langle m\rangle)\eta \end{array} \tag{5.8}$$

Figure 23: Plate Parameter Variation for Epoch 1900.

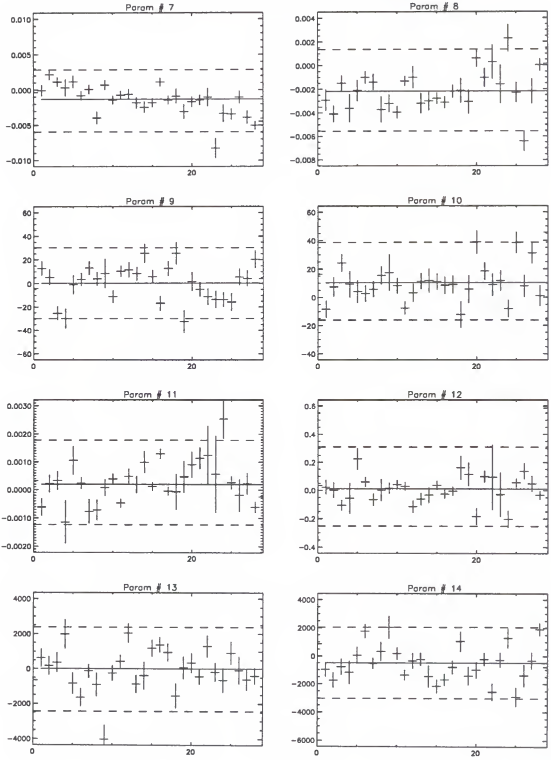


Figure 24: Plate Parameter Variation for Epoch 1956.

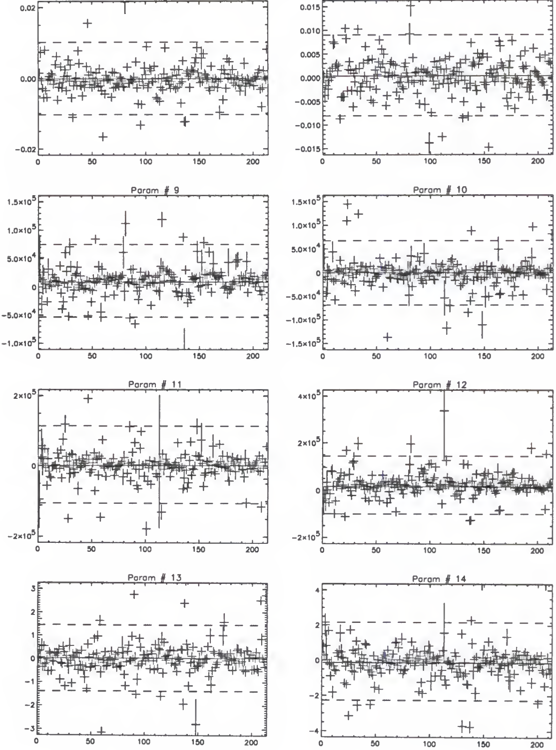
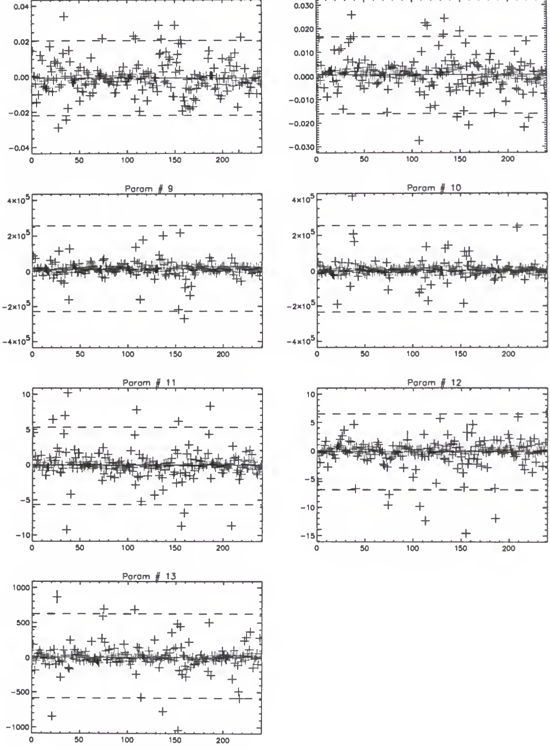


Figure 25: Plate Parameter Variation for Epoch 1992.



with constrained terms in the following:

$$\begin{matrix} (\xi^2+\eta^2) & 0 \\ 0 & (\xi^2+\eta^2). \end{matrix} \tag{5.9}$$

For epoch 1956,

$$\begin{matrix} (m-\langle m\rangle)^2\xi & 0 \\ 0 & (m-\langle m\rangle)^2\eta \end{matrix} \tag{5.10}$$

with constrained terms in the following:

$$\begin{matrix} (m-\langle m\rangle)\xi(\xi^2+\eta^2) & 0 \\ 0 & (m-\langle m\rangle)\eta(\xi^2+\eta^2) \end{matrix} \tag{5.11}$$

Finally for epoch 1992,

$$\begin{matrix} (m-\langle m\rangle)^2\xi & 0 \\ 0 & (m-\langle m\rangle)^2\eta \end{matrix} \tag{5.12}$$

with constrained terms in the following:

$$\begin{matrix} (m-\langle m\rangle)\xi(\xi^2+\eta^2) & (m-\langle m\rangle)\eta(\xi^2+\eta^2) \\ (m-\langle m\rangle)\eta(\xi^2+\eta^2) & (m-\langle m\rangle)\xi(\xi^2+\eta^2) \end{matrix} \tag{5.13}$$

Overlap Variances

After the overlap has been carried out the stellar positions will have a error from equation 3.56:

$$\epsilon = (s\mathbf{B}\beta + \Xi a - d). \tag{5.14}$$

The square of these quantities will provide the variance of the star's position. This variance will still reflect inherent problems in finding stellar positions. As we have discussed the positional precision of a star is related to its magnitude and to its proximity to the center of the plate. The positions of the stars at the extremes in magnitude (brightest, faintest) and plate position (in the corners and edges) will not be determined as precisely as those in the middle of the range / plate. However, after one run we can find how the variance is correlated with the stellar parameters, position and magnitude.

In figure 26 and 27 we plot the variances of a nine-constant fit, that includes both first order magnitude, and origin distance for the AC material. The variances are plotted vs first second and third order magnitude, coordinate and origin distance. We see positive correlations, and can fit a least square polynomial to all the system variances and then re-run the overlap including the predicted variance rather than the constant variance we use in the first run. It is important to return to the original estimates of the stellar equatorial positions; otherwise the variance information is not 'new' information. If we do not return to the original estimates the re-computation will just reconfirm that our input has the predicted variance variation.

This ends the discussion on choosing the model. In all solutions the covariance matrix, correlation matrix, matrix efficiency, and condition number are calculated and checked to make sure the system is still stable.

Figure 26: Nine-Constant Overlap Variances

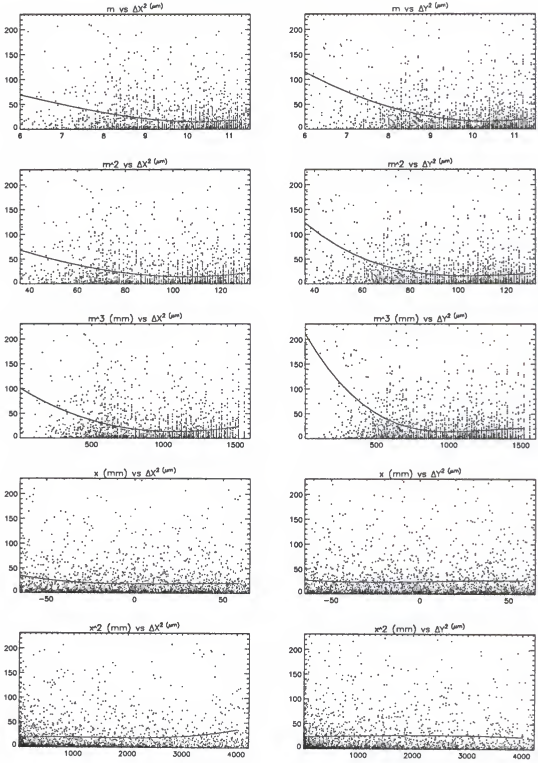
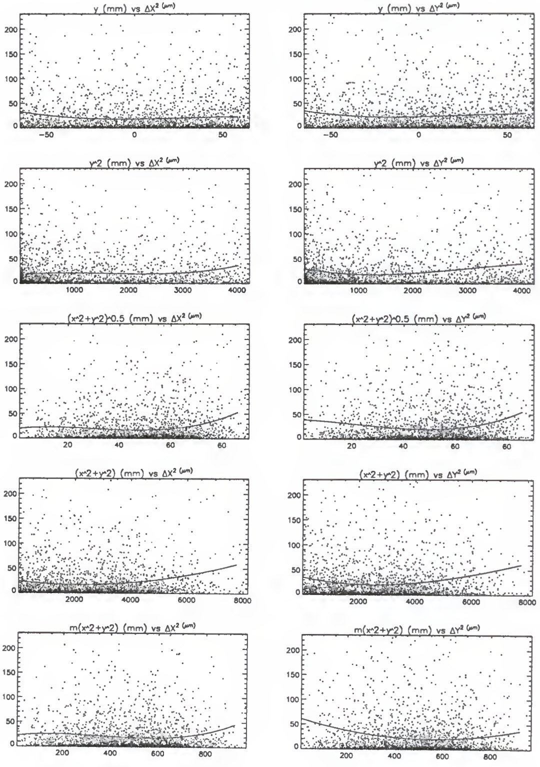


Figure 27: Nine-Constant Overlap Variances



CHAPTER 6
PROPER MOTIONS

Finding Proper Motions

The proper motion of a star is defined as the time derivative of a star's equatorial coordinate in an (ideally) inertial frame. In this study the nearly inertial frame is provided by the Astrographic Catalogue of Reference Stars using the B1950 (Besselian Date 1950) positions. This means all three epochs are, within the accuracy of the reference material, in the FK4 approximation to an inertial reference frame.

To keep the plate parameters and centers as close as possible to those published when the plates were exposed we precessed, using IAU constants, all the reference material (and by default the inertial frame) to 1900 for the first epoch. After we found first epoch positions we then precessed them back to B1950, the epoch of the reference catalogue and the second and third observations.

The difference in the stellar positions, after this procedure, between the three epochs is due to the proper motion of the respective stars. The relationship of positions (α,δ) at any given date (*t*), with the stellar positions (α_o, δ_o) and the proper motions (μ) at the date of observation (t_o), are

$$\alpha = \alpha_o + \mu_\alpha(t - t_o) \qquad \delta = \delta_o + \mu_\delta(t - t_o). \tag{6.1}$$

Note that the proper motion, μ, has no time suffix; this is because we assume uniform motion and therefore $\mu = \mu_o$.

To obtain the best estimates for the position and proper motion we should find these quantities at the position variance weighted epoch. At this point the position and

proper motion estimates are uncoupled and therefore uncorrelated. In this example the three position estimates have a variance ratio of approximately 4:1:1 in 1900:1956:1992 respectively. The variance weighted average epoch is then (1900+(4*1956)+(4*1992))/9 $\approx$ 1965.

However, the plates were not all exposed at the same time. It has already been pointed out that the AC material was exposed over a period of 10 years and the second epoch McCormick observations took over a year. In the overlapping plate method we assume that a star on different plates (within the same epoch) has the same position. However, because of the time difference between exposures this will not be the case.

One way to alleviate the problems the above assumption could cause is the following; after the overlap, rather than using the β value to correct the input and get one mean weighted position, actually use the plate parameters to find the position of a star on each plate. The strength of the overlap enters into this estimate of the star's position via the plate parameters. However, the actual star position is no longer confined to just the average for that epoch, it can rather assume the value derived from each plate along with the exposure date of that plate. So if two plates that overlap have a date of exposure difference of ten years, then the star's position will be allowed to reflect this.

There is a more rigorous method. Eichhorn (1993) has derived a method that allows complete generality in the position by introducing the proper motion as an additional parameter at the beginning of the reduction. In this reduction the time difference is most acute in the AC material, which is also the material with the lowest positional precision. The increased computational effort required to incorporate Eichhorn's new method is therefore unwarranted and the above approximate approach is sufficient.

Therefore we find the proper motion from a least squares analysis of all occurrences of a star in every exposure. Following the discussion in chapter two we first set up the condition equations,

$$\mathbf{F} = \begin{pmatrix} \alpha-(\alpha_1+\mu_\alpha(1965-t_1)) \\ \delta-(\delta_1+\mu_\delta(1965-t_1)) \\ \alpha-(\alpha_2+\mu_\alpha(1965-t_2)) \\ \delta-(\delta_2+\mu_\delta(1965-t_2)) \\ \vdots \\ \alpha-(\alpha_n+\mu_\alpha(1965-t_n)) \\ \delta-(\delta_n+\mu_\delta(1965-t_n)) \end{pmatrix} = \begin{pmatrix} \alpha-\alpha_1+\mu_\alpha(t_1-1965) \\ \delta-\delta_1+\mu_\delta(t_1-1965) \\ \alpha-\alpha_2+\mu_\alpha(t_2-1965) \\ \delta-\delta_2+\mu_\delta(t_2-1965) \\ \vdots \\ \alpha-\alpha_n+\mu_\alpha(t_n-1965) \\ \delta-\delta_n+\mu_\delta(t_n-1965) \end{pmatrix} \tag{6.2}$$

where $\alpha, \mu_\alpha, \delta,$ and μ_δ are the target quantities, $\alpha_1, \alpha_2, ...\alpha_n$ and $\delta_1, \delta_2, ...\delta_2$ are the 'observations' from the plate solutions at dates $t_1, t_2, ...t_3$, respectively.

Following equations 2.12 and 2.13, we define the two quantities: $\mathbf{A} = \left(\frac{dF}{da}\right)_{x=x_o,a=a_o}$ and $\mathbf{X} = \left(\frac{dF}{d\epsilon}\right)_{x=x_o,a=a_o}$ then the corrections to the parameters α (equation 2.20) are

$$\alpha = -\left[\mathbf{A^T}(\mathbf{X}\sigma\mathbf{X^T})^{-1}\mathbf{A}\right]^{-1}\mathbf{A^T}(\mathbf{X}\sigma\mathbf{X^T})^{-1}\mathbf{F_o}. \tag{6.3}$$

As in the overlap we have the simplification that each equation contains only one observation and therefore $\mathbf{X} = -\mathbf{I}$. The matrix $\mathbf{A}$ is

$$\mathbf{A} = \frac{\partial \mathbf{F}}{\partial(\alpha, \delta, \mu_\alpha, \mu_\delta)} = \begin{pmatrix} 1 & 0 & \tau_1 & 0 \\ 0 & 1 & 0 & \tau_1 \\ 1 & 0 & \tau_2 & 0 \\ 0 & 1 & 0 & \tau_2 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & \tau_n & 0 \\ 0 & 1 & 0 & \tau_n \end{pmatrix} \tag{6.4}$$

where τ is the time difference t_i–1965. Assuming a first approximation of zero for the four unknowns, then $\mathbf{F_o}$ is

$$\mathbf{F}_o = \begin{pmatrix} -\alpha_1 \\ -\delta_1 \\ -\alpha_2 \\ -\delta_2 \\ \vdots \\ -\alpha_n \\ -\delta_n \end{pmatrix}. \tag{6.5}$$

where τ is the time difference t_i–1965. Assuming a first approximation of zero for the four unknowns, then $\mathbf{F_o}$ is

$$\mathbf{F}_o = \begin{pmatrix} -\alpha_1 \\ -\delta_1 \\ -\alpha_2 \\ -\delta_2 \\ \vdots \\ -\alpha_n \\ -\delta_n \end{pmatrix}. \tag{6.5}$$

The parameters simplify to $\alpha = -\left[\mathbf{A}^{\mathbf{T}}\sigma^{-1}\mathbf{A}\right]^{-1}\mathbf{A}^{\mathbf{T}}\sigma^{-1}F_o$. The first term simplifies to

$$\left[\mathbf{A}^{\mathbf{T}}\sigma^{-1}\mathbf{A}\right] = \begin{pmatrix} \sum\sigma_\alpha^{-1} & 0 & \sum\sigma_\alpha^{-1}\tau & 0 \\ 0 & \sum\sigma_\delta^{-1} & 0 & \sum\sigma_\delta^{-1}\tau \\ \sum\sigma_\alpha^{-1}\tau & 0 & \sum\sigma_\alpha^{-1}\tau^2 & 0 \\ 0 & \sum\sigma_\delta^{-1}\tau & 0 & \sum\sigma_\delta^{-1}\tau^2 \end{pmatrix} \tag{6.6}$$

where all terms are summed over all observations. This is a (4×4) matrix. The second term simplifies to the (4×1) vector:

$$-\left[\mathbf{A}^{\mathbf{T}}\sigma^{-1}\mathbf{F_o}\right] = \begin{pmatrix} \sum\sigma_\alpha^{-1}\alpha \\ \sum\sigma_\delta^{-1}\delta \\ \sum\sigma_\alpha^{-1}\tau\alpha \\ \sum\sigma_\delta^{-1}\tau\delta \end{pmatrix}. \tag{6.7}$$

The parameter errors will be

$$\left(\mathrm{diag}\left[\mathbf{A}^{\mathbf{T}}\sigma^{-1}\mathbf{A}\right]^{-1}\right)^{\frac{1}{2}} \tag{6.8}$$

and the observation errors (residuals) will be $\varepsilon = (\mathbf{F_o} - \mathbf{A}\alpha)$.

Results

Table (9) lists the calculated position and proper motion for the epoch 1965 equinox 1950B. All stars that were found in at least two epochs are included. Of the original 2600 stars from the Astrographic Catalogue Plates 1678 were found in at least one more epoch. The others are predominantly in the edge regions not covered in the McCormick Observations.

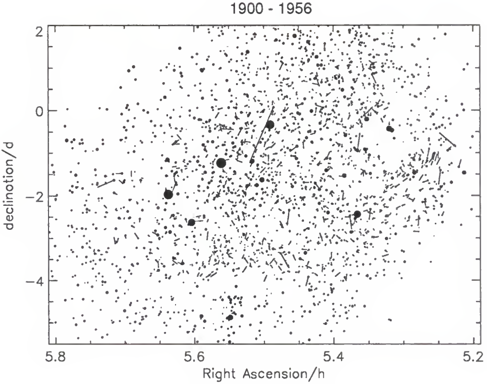
From the first epoch there were 6285 observations, from the second epoch there were 35,378 and from the last epoch 33,352. The positional variances were approximately in the ratio 4:1:1 for the 1900:1956:1992 epochs respectively. This is expected if one considers the higher measurement precision and longer focal length in the second and third epochs.

Table (6) lists the mean error and standard deviation of the formal errors in the final solution.

Table 6: Final Solution Statistics

Quantity	Right Ascension	Declination
Position Error Mean, Standard Deviation. ″	0.0142, 0.013	0.050, 0.046
Proper Motion Error Mean, Standard Deviation. ″/century	0.0058, 0.0035	0.0217, 0.0139

One way to internally check the result is to see how the results of comparing the 1900 material to only one of the other two epochs differs from that of the least squares answer. Figure 28 presents the results of a very simple calculation that takes the difference in the stellar position at the two epochs and extends it out by a factor of 200. The star is at the tail of the proper motion. As one can see if one takes into account the fact that the epoch difference is almost twice as much for the 1900–1992 than the 1900–1956 observations, these graphs are qualitatively the same.



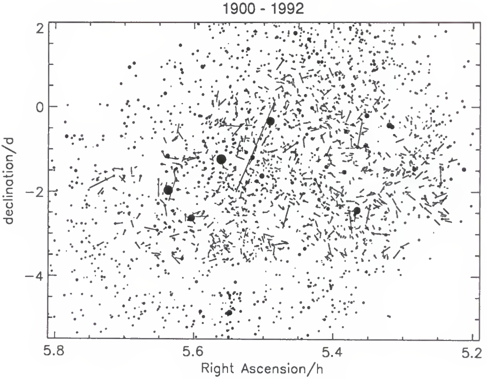


Figure 28: Proper Motions Using Just Two Epochs

Figure 29 shows the least squares solution for proper motions over a 10,000 year period. These also reflect the general pattern seen in the last two graphs. The high proper motion star in the middle of the picture is HD 36443 a 8.6 magnitude G5 star that is not believed to be part of the association.

Note that none of the belt stars are included because they were not measured in the Astrographic Catalogue. The positions for stars as bright as these are very difficult to determine from photographic plates because of the sizes of their images. For informational purposes, their positions are marked by their names: ϵ, δ , and ζ.

In the second graph in figure 29 we show the proper motion diagram of our sample. If the association members had a significant common proper motion, then they would group together in the same portion of the diagram. The mean proper motion of this group would then define the mean proper motion of the association. However, an examination of the diagram shows no such structure. and the membership is not easily obtainable from just the proper motions. After further analysis of the proper motion components, we felt that membership determination using proper motions would require an examination beyond the scope of this dissertation.

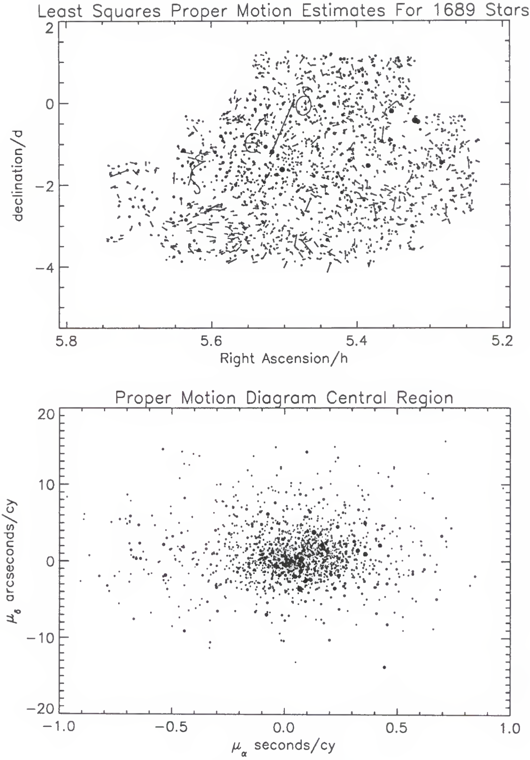


Figure 29: Proper Motion Plots Final Result

A good way to judge the accuracy of our results is to compare the positions and proper motions with those of the Astrographic Catalogue of Reference Stars. This is still an internal check because these reference stars were used as our original input. It may however highlight systematics. Figure 30 shows the proper motions of the reference stars in our region taken directly from the ACRS. Again there is good qualitative agreement between this figure and the first graph in figure 29.

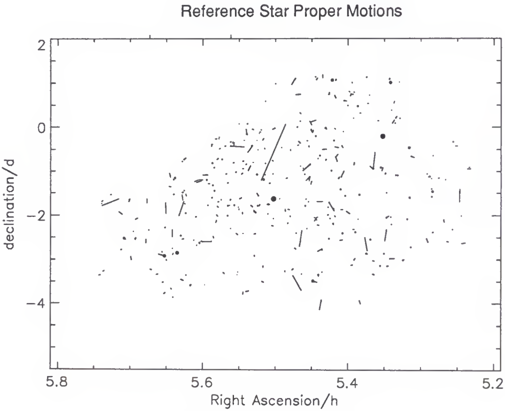


Figure 30: Proper Motions of Reference Stars in our Sample

By finding the difference between the reference stars positions and proper motions in 1965 and our calculated positions, we can find the 'residuals' of our results. This can also be done by comparing our results with AGK3U, a updated version of the

AGK3 catalogue. We have 341 stars overlapping with the ACRS and 206 stars with the AGK3U. Figure 31 shows the histograms of the difference between our positions / proper motions and these two catalogues. The curve represents a best fit gaussian.

The striking feature of these plots is that the residual in declination is distributed over two times the range of the right ascension residual. There could be a number of intrinsic reasons for this which may be related to the actual observations and measuring. For example the observations were all hand guided, and the guiding corrections made during an actual exposure were nearly always in right ascension. The McCormick telescope is equatorial mounted telescope, thus if the telescope had an axes alignment problem, then there would be a drift in declination. This may be imperceptible to the observer, but would lead to a systematic guiding error in the declination direction and hence a larger variance in that coordinate. Another possibility for the larger declination residual may be due to the measuring machine. The plates were all placed on the machine so that the x and y coordinates always had the same orientation with respect to the measuring axes. If there was a continual slip in the y direction this would also cause a large variance in the declination because this axis during observation would be aligned with that coordinate. A re-analysis of both observing device / procedure and the measuring device / procedure is needed to distinguish between these and other possibilities.

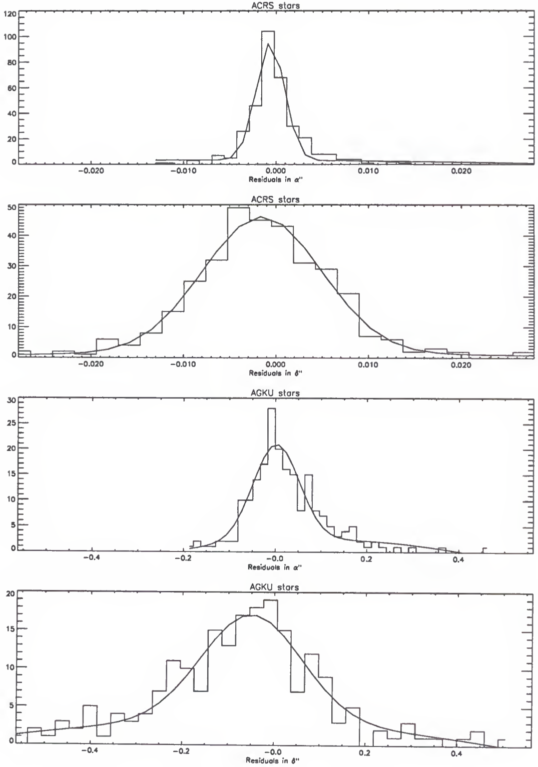


Figure 31: Differences between these Results and External Catalogues

This ends our discussion of determining the proper motions in Orion. The results, data and programs will be available by anonymous file-transfer-process from astro.ufl.edu. Next we will discuss the use of these proper motions and the options for future work.

Using the Proper Motions

Historically proper motions of associations and clusters have been used to find the expansion age and kinematical properties of the association / cluster. These proper motions need to be included with spectroscopic and radial velocity data to establish membership criteria. Once the members are determined, the expansion age and other astrophysically interesting properties such as the H-R diagram, initial mass function, and kinematical properties can be found.

As stated before, it is beyond the scope of this investigation to examine the membership criteria, but we shall use our data to find what the simple linear expansion age would be assuming the association is expanding. We have adopted the method developed by Blaauw (1952) to determine a linear expansion. He showed that should an expansion exist and provided that; 1) the present dimensions of the group are larger than the original dimensions and 2) all the motions in the group started at the same time and are uniform, then there should be a linear relationship between the component of the proper motion and the corresponding coordinate. One must also allow for perspective contraction or expansion caused by the relative motion of the Sun.

Using this method Lesh (1968) found an expansion age of 4.5 billion years for the northwest region of I-Orion. This examination was based on 16 stars which were

not found in our sample. However there have been two other studies (Warren and Hessler, 1977 and Gieseking, 1983) that have found possible members which are within our data set. We will adopt their membership choices and use our proper motion components to investigate the expansion. The Warren and Hessler study used primarily a photometric determination of membership, while the Gieseking study was a radial velocity investigation of highly probably members. We will consider these as studies of the Orion members using spectroscopic and radial velocity criteria, respectively.

Spectroscopic Criteria

There is a sizable amount of spectroscopic information available, however, most of it is concerned with the bright stars. The above mentioned study by Warren and Hessler in 1977 (hereafter WH) collected all the spectroscopic criteria available and combined this with *ubvy* observations to produce a membership ranking for 504 stars in the association, of which 107 overlap with this study. The stars are all ranked on their probability of membership in radial velocity, proper motion and photometric properties as a, b or c. An 'a' represented a high probability of membership, while a 'c' represented a low probability.

In figure 30 we plot the proper motion components vs. their respective coordinates. A variance weighted least squares fit to the data is shown. The first two graphs represent very high probability stars which were classified as 'a' by WHs' membership criteria in all three respects; proper motion, radial velocity, and photometry. If the straight line fit has a positive slope, then this indicates an expansion; while a negative slope indicates a contraction. As one can see, the slopes are not strongly positive, they are either basically flat or negative. The slopes and the standard deviations of the least squares fit are shown

in table 7. None of the slopes are above the standard deviation, so their significance must be questioned. However, should these slopes actually signify true kinematic properties of the association, then in actual fact the association is *contracting*.

It must be noted that this is a very simplified analysis that can be questioned on many different grounds. For example the membership criterion was adopted wholesale from WH; and by using more recent observations, better membership criteria may be possible. We have also not allowed for a distance gradient as proposed by WH, because there was not enough available distance information for this small sample size to make this consideration useful. Also, there is a possibility that galactic rotation might distort the proper motions. All of these considerations were considered outside the scope of this investigation but will be interesting to consider as future projects.

Table 7: Least Squares Slopes using WH Criteria.

Coordinate	Number of stars in sample	Slope deg^{-1} $year^{-1}$	Standard Deviation
Right Ascension, all criteria =a.	26	-0.0049	0.8881
Declination, all criteria =a.	26	-0.2581	0.7999
Right Ascension, photometric criteria =a.	103	-0.0032	1.1089
Declination, photometric criteria =a.	103	-0.0598	0.7977

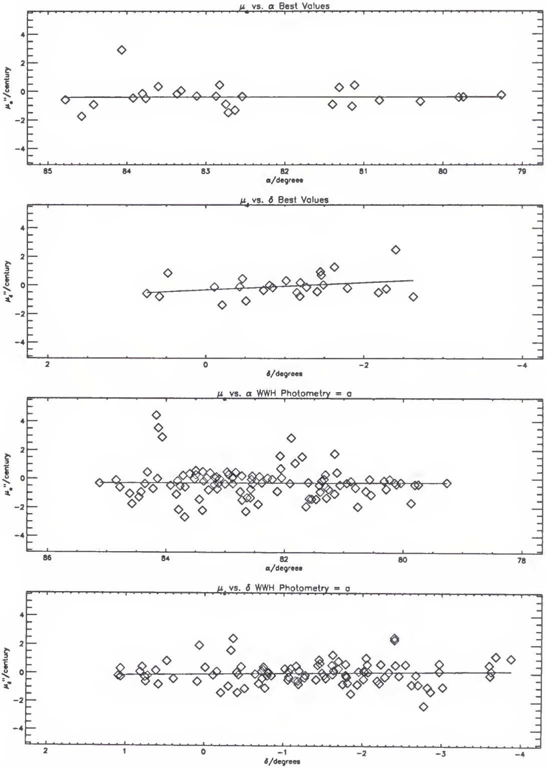


Figure 32: Linear Relations between μ_α and α and μ_δ and δ using WH Membership Criteria

Radial Velocity Criteria

Radial velocities can also be used to determine association membership. However, measuring radial velocities is very time consuming and to date only a handful of the stars in this study had their radial velocities measured. Due to the inherent observational difficulty of measuring radial velocities, these are also predominantly bright stars. If radial velocities from different observatories are used, then an investigation of individual observatory corrections would be required. Therefore, we will adopt the results of Gieseking's (1983) study of 66 of high probability Orion Ib members. Gieseking felt there was a bimodal distribution in the group's radial velocity distribution centered on 4.5km/s (relative velocity).

Of the 66 stars in his study 50 are also in this study. The expansion was again investigated by plotting the proper motion components against their respective coordinates, as shown in figure 33. In these graphs the symbols '•' represent the stars with a velocity below 4.5 km/s and '+' those greater than 4.5 km/s. The three lines represent a least squares best line fit to the stars with velocities less than 4.5 km/s (dotted line), those with velocities greater than 4.5 km/s (dashed line), and all the stars in the sample (solid line). The dashed circles represent the variance of each star's proper motion measurement. The slopes and the standard deviations of the least squares fit are shown in table 8.

Again nothing conclusive can be stated because the standard deviation are greater than the slope of the positions. However there is a consistent expansion in both coordinates and it is possible to calculate an expansion using these slopes.

Because Orion is receding almost directly in the direction of the solar antapex we

can assume that the recession velocity will give an apparent contraction (Blaauw 1952, Lesh 1968) of

$$\frac{\pi}{180}\frac{V_r}{4.74}\frac{1}{r} \tag{6.9}$$

where V_r is the velocity of recession and r is the distance. Using a recession velocity of 20.8 kms^{-1} and a distance of 400 pc we arrive at a contraction due to recession of 0.019 degrees / century. Using a weight least square slope of all the observations in both coordinates and adding in this recessional term we find an expansion age of 1.22 $\times$ 10^6 years with a standard deviation of 200%. This is basically consistent with what one would expect but in fact totally inconclusive.

Table 8: Least Squares Slopes using Gieseking Criteria.

Coordinate	Number of stars in sample	Slope deg^{-1} year^{-1}	Standard Deviation
Right Ascension, all criteria =a.	55	0.468	0.936
Declination, all criteria =a.	55	0.145	0.756
Average of both slopes	55	0.277	0.582

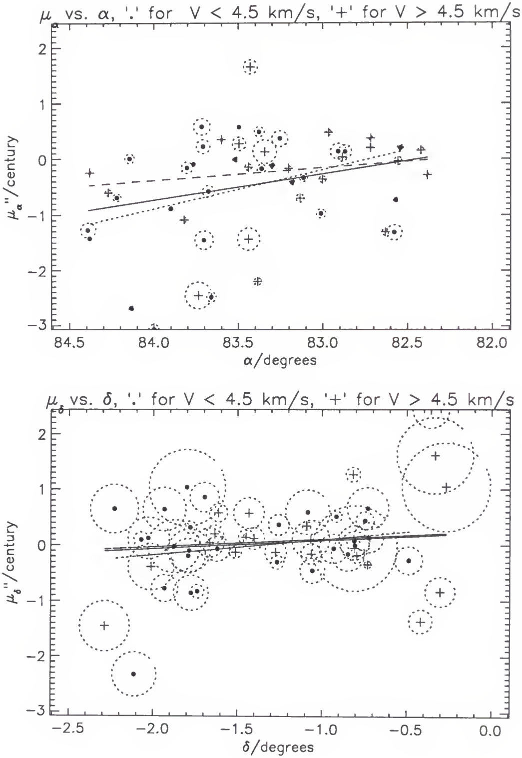


Figure 33: Linear Relations between μ_α and α and μ_δ and δ using Gieseking Membership Criteria

It appears that using the best currently published membership criteria and a simplified analysis that no conclusive proof of either expansion or contraction can be obtained. This will be discussed again in the section on future work.

Future Work

The proper motions estimates derived in this investigation may be improved with more information, primarily color and additional observations. The models we checked never included color terms because we did not have this information for the field stars. When the Guide Star Catalogue II is available this will change as this catalogue is complete down to 12th magnitude, in combination with the first catalogues will give two color magnitudes. This is obviously not as useful as spectral types but will suffice for testing the material for any color correlations. In addition to providing colors, the two guide star catalogues will provide two extra epoch observations. The estimates will not be as accurate because the guide star has a high error budget and used astrometrically inferior Schmidt plates, but they will still provide valid estimates of the stellar positions.

The reduction procedure used in this investigation can be improved upon. Some of the parameters, notably the focal length, are very strong candidates for stochastic constraints. This will allow for less variance in the final solution. We could also include proper motions from step one. Rather than reducing the three epochs separately one can reduce them simultaneously, the proper motions will then be parameters from that reduction. This will provide for more internally consistent estimates of all the stellar parameters. Finally, the model we have chosen can probably be improved with more investigation. This is always an option but the expected gains are less than those from incorporating one of the improvements above.

This study provides consistent proper motions estimated for a large sample of stars in the region of the Orion association. These need to be investigated in conjunction with all the currently available spectroscopic and radial velocity criteria to determine as large a membership sample as possible which is free from contamination. This sample should then be examined with the methods outlined above, using space velocities, and using proper motions in galactic coordinates. These examinations should be applied to the association as a whole and to the respective subgroups individually. This study has shown that when considering previously determined member, nothing conclusive can be drawn about the expansion or contraction of the association.

Table of Results

Table 9 lists the proper motions and positions for stars found in the Astrographic Catalogue and at least one of the McCormick Epochs. The epoch of the position is 1965 and the equinox is 1950B. Listed are:

1. AC # — the running number for the star in the 1900 AC plates
2. ACRS # — the ACRS number if the star is also part of the reference catalogue
3. N — the number of images found for the star
4. m — the magnitude from the AC.
5. α — the right ascension in hours, minutes and seconds
6. ϵ_α — the right ascension position error in seconds
7. δ — the declination in hours, arcminutes and arcseconds
8. ϵ_δ — the declination position error in seconds
9. μ_α, — the right ascension proper motion in seconds / century
10. $\epsilon\mu_\alpha$ — the right ascension proper motion error in seconds / century
11. μ_δ — the declination proper motion in arcseconds / century
12. $\epsilon\mu_\delta$ — the declination proper motion in arcseconds / century

Table 9: Positions and Proper Motions for Stars Found in at least two Epochs.

AC#	ACRS#	N	m	α	$\epsilon\alpha$	δ	ϵ_δ	$\mu\alpha$	$\epsilon\mu\alpha$	μ_δ	$\epsilon\mu_\delta$
68	0	3	9.95	5 19 13.93408	.03550	+1 0 40.9671	.0194	-.058170	0.008584	-0.53104	0.00784
74	0	3	10.20	5 19 31.10795	.05371	+1 5 37.1623	.2219	0.081102	0.017674	-2.20061	0.06161
77	0	5	9.70	5 19 41.79936	.07090	+1 2 58.5744	.1648	-.042841	0.020168	-1.16296	0.03743
79	0	3	9.85	5 19 46.27236	.03869	+1 6 13.3381	.1206	-.043035	0.007494	-0.62094	0.03685
80	48153	9	8.97	5 19 47.61690	.01832	+1 1 27.2588	.0604	0.002983	0.011165	-1.41879	0.05350
96	48268	5	6.50	5 20 30.62946	.03574	+1 0 41.3142	.2416	0.004181	0.010572	0.54709	0.06697
101	48278	8	7.00	5 20 35.12362	.01805	+1 5 36.6548	.1017	-.017050	0.007497	-0.17023	0.04773
102	48286	10	9.23	5 20 37.48538	.02963	+1 5 53.2111	.0886	-.056481	0.009918	-0.57310	0.04804
106	0	3	10.95	5 20 46.64333	.03495	+1 5 6.2418	.2004	-.010492	0.006857	-3.66295	0.08001
108	0	8	8.40	5 20 47.85081	.01834	+1 0 34.5679	.0697	-.044325	0.009132	-0.45414	0.03454
116	0	3	10.80	5 20 59.12528	.05063	+0 59 19.7747	.1428	-.223118	0.011895	-3.21683	0.04659
120	0	3	11.30	5 21 10.85757	.14878	+1 4 13.0041	.2538	0.027294	0.026889	-1.85641	0.05028
126	0	3	10.80	5 21 20.67585	.06460	+0 58 49.4437	.0996	0.107896	0.018525	-3.58878	0.02181
151	0	4	10.43	5 21 56.53044	.03188	+1 1 7.6842	.1344	0.024072	0.008869	-0.36952	0.04102
162	48618	6	8.97	5 22 28.37906	.03458	+1 8 30.6276	.1031	-.070034	0.013122	-0.63611	0.03003
170	0	7	9.50	5 22 37.68600	.03865	+1 4 4.4259	.1554	-.044368	0.008342	-0.37868	0.03301
171	0	9	10.07	5 22 42.33728	.02421	+1 0 55.2642	.0904	-.053907	0.004215	-0.84021	0.02266
172	0	9	11.03	5 22 45.22110	.02645	+0 58 29.3114	.1045	-.068345	0.007751	0.27755	0.04263
174	0	16	9.32	5 22 46.26998	.02627	+1 3 9.6402	.0984	-.019243	0.009272	0.19168	0.03290
175	0	4	11.03	5 22 53.91569	.04992	+1 6 8.0598	.1085	0.047747	0.013189	1.02172	0.03609
187	0	6	9.93	5 23 32.16350	.02785	+1 3 7.1046	.0938	0.005637	0.009999	-0.33212	0.04842
189	0	4	10.83	5 23 34.08800	.01799	+0 59 46.2554	.1997	0.020815	0.014084	0.80824	0.04717
191	48831	10	8.37	5 23 41.72820	.03070	+1 0 58.7981	.1848	0.016753	0.010589	-0.23118	0.04330
194	0	4	10.47	5 23 49.15570	.05234	+1 1 19.4355	.2069	0.053085	0.012067	-0.29158	0.03912
206	0	3	10.75	5 24 22.77490	.03392	+1 4 41.7385	.0924	-.061851	0.007405	0.29778	0.02495
215	0	3	10.70	5 24 44.96618	.05353	+1 3 33.5582	.2165	-.037427	0.016466	-0.44204	0.04901
216	0	6	8.13	5 24 47.24583	.02067	+0 58 35.0280	.0986	-.051702	0.008308	-0.32725	0.03353
219	49082	10	6.73	5 25 1.86486	.01964	+1 4 0.3772	.1052	-.047692	0.007888	0.34429	0.04164
222	0	4	10.53	5 25 10.95558	.03893	+0 59 19.7004	.1380	-.017377	0.015141	-1.12098	0.04639
226	49149	9	6.63	5 25 19.22907	.02241	+1 3 52.1760	.1403	-.001843	0.009647	-0.24110	0.04320

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
244	49306	23	7.40	5 26 6.93339	.01818	+1 11 16.2039	.0776	0.051827	0.015946	-1.76019	0.07868
245	0	21	9.50	5 26 8.94222	.01454	+1 9 16.1279	.0711	-.043133	0.002821	0.37934	0.03141
256	49382	43	8.47	5 26 31.30183	.00694	+1 6 55.5800	.0344	0.051574	0.003616	-1.63925	0.01499
261	49411	44	8.60	5 26 43.50786	.00696	+1 8 36.0923	.0326	-.006194	0.004160	0.42866	0.01646
310	0	20	10.63	5 26 57.87720	.00721	+1 3 37.4421	.0360	-.087338	0.003955	-0.10702	0.02453
321	0	46	8.47	5 27 27.84206	.00617	+1 4 16.9875	.0233	-.059184	0.004496	-0.27105	0.01907
328	0	20	10.80	5 27 59.07495	.01043	+0 57 56.0610	.0456	-.134613	0.007973	-2.45984	0.03179
331	49669	52	9.40	5 28 6.20351	.00449	+1 4 55.5307	.0170	-.016287	0.002973	-0.33424	0.01111
336	49724	48	8.67	5 28 21.12861	.00575	+1 6 35.6099	.0223	0.194601	0.003917	-5.25666	0.01440
343	0	29	9.15	5 28 29.11308	.00939	+1 9 4.0150	.0424	-.047721	0.003926	-0.88455	0.00768
352	0	51	9.00	5 28 55.49470	.00916	+1 9 9.8006	.0366	-.005020	0.005326	-0.76325	0.02964
359	0	44	9.97	5 29 37.08858	.00833	+1 1 21.7016	.0248	0.074595	0.005724	-2.84136	0.01560
360	0	30	9.65	5 29 37.57901	.01078	+1 15 14.3206	.0692	0.039964	0.004661	-0.87868	0.03553
363	0	17	11.10	5 29 49.99838	.01076	+1 5 5.9998	.0396	0.054650	0.003276	1.17931	0.01518
366	50008	38	8.87	5 29 57.64954	.00919	+0 59 19.3547	.0484	0.050079	0.008326	-0.00517	0.04590
370	0	18	10.17	5 30 14.78503	.01002	+0 58 44.5292	.0341	-.027678	0.005717	-0.97255	0.01850
373	0	18	11.33	5 30 29.52906	.01274	+0 59 28.5710	.0368	0.014420	0.007747	-1.91073	0.02249
383	0	18	10.33	5 31 14.34948	.01059	+1 4 55.6595	.0424	-.065021	0.006178	-0.89951	0.02884
384	0	44	8.73	5 31 19.66120	.00913	+1 0 0.6351	.0337	-.043809	0.008379	-0.73980	0.02938
388	0	15	9.85	5 31 28.18462	.01340	+1 13 44.2710	.0564	-.050276	0.004724	-0.90168	0.04679
391	0	17	10.65	5 31 39.25057	.02276	+1 3 25.7021	.0481	-.030796	0.004633	-0.81440	0.03325
394	0	15	10.20	5 31 52.46613	.01369	+1 10 24.1720	.0506	-.136394	0.008926	-0.57845	0.02983
399	0	18	10.87	5 32 17.39397	.01579	+1 0 32.2380	.0780	0.007717	0.007500	-0.73139	0.02330
400	0	35	8.45	5 32 17.64017	.01121	+1 9 30.0956	.0356	-.016889	0.007463	-0.60736	0.01886
401	0	18	10.87	5 32 26.06686	.02079	+1 2 44.5002	.0507	-.026430	0.006880	-0.86941	0.02559
509	0	13	9.60	5 19 14.21160	.01451	+0 9 54.1842	.0709	-.008436	0.006480	-0.28733	0.04896
510	0	12	11.20	5 19 19.23164	.02648	+0 4 6.7215	.0591	0.181697	0.012014	-2.56401	0.04062
511	0	5	9.15	5 19 39.26701	.02411	+0 54 45.9597	.1400	-.098739	0.006689	-0.69237	0.03831
512	0	11	9.25	5 19 41.30256	.02065	+0 30 55.4904	.0355	-.014410	0.011608	-0.51568	0.01472
513	48134	49	8.90	5 19 41.34586	.01259	+0 11 48.0731	.0574	0.002636	0.012833	0.56174	0.05568
514	0	9	10.90	5 19 43.64541	.02158	+0 33 45.9160	.0746	-.013271	0.008224	-0.18976	0.05327
515	0	5	9.20	5 19 45.68715	.03592	+0 56 1.3999	.1050	-.001942	0.010648	-0.71026	0.03072
516	0	9	10.95	5 19 47.96433	.02753	+0 20 23.8612	.0560	-.060881	0.008739	-1.15446	0.01661

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
517	0	11	10.30	5 19 49.49039	.01925	+0 7 40.4544	.0710	0.088018	0.015468	-2.12300	0.03536
518	0	14	11.15	5 19 56.72180	.01177	+0 23 52.8664	.0310	-.125358	0.004650	-2.36121	0.01694
519	514189	11	10.70	5 20 0.02231	.02379	+0 14 37.4075	.0848	0.004361	0.016344	-0.79880	0.06016
520	48198	57	7.57	5 20 9.24185	.00600	+0 5 33.3949	.0242	-.018098	0.003912	-0.58763	0.01255
521	0	17	10.70	5 20 10.58007	.00940	+0 29 3.0141	.0261	-.042526	0.003786	-0.45405	0.00960
522	48205	41	8.75	5 20 10.93718	.00653	+0 35 24.8670	.0215	0.041457	0.002844	-1.14069	0.01062
523	48208	64	8.65	5 20 11.57245	.00493	+0 9 46.7732	.0200	0.093368	0.004129	-2.60099	0.01165
524	0	27	11.10	5 20 15.73436	.00953	+0 10 6.1486	.0248	0.054514	0.004396	0.31758	0.01400
525	0	17	8.60	5 20 18.12514	.01235	+0 53 10.7545	.0461	-.032316	0.005126	0.88525	0.01731
526	0	11	10.70	5 20 22.43469	.01186	+0 56 13.4367	.0351	-.050596	0.001915	0.04638	0.00749
527	0	18	11.25	5 20 24.47964	.01257	+0 36 1.3081	.0296	0.098268	0.005454	-1.71029	0.00798
528	0	17	11.20	5 20 27.92381	.01508	+0 35 34.9392	.0576	0.061631	0.005447	0.05601	0.01308
529	48253	41	7.45	5 20 27.74068	.00863	+0 23 34.1966	.0397	-.024928	0.003664	-0.38985	0.01595
530	0	23	7.80	5 20 29.32118	.01002	+0 51 53.4356	.0484	-.067036	0.003790	0.47868	0.01880
531	0	21	9.60	5 20 31.08095	.00920	+0 33 48.0401	.0513	-.084491	0.003396	-3.09946	0.02006
532	0	17	9.45	5 20 37.62007	.01251	+0 52 1.9337	.0592	-.047560	0.004747	0.67168	0.02280
533	0	31	10.70	5 20 38.08984	.00944	+0 1 18.4880	.0297	-.123523	0.005095	-3.34065	0.01333
535	0	27	10.85	5 20 47.27294	.00935	+0 11 45.4915	.0265	0.014988	0.003933	-1.00016	0.00637
536	48326	39	7.85	5 20 54.43502	.01005	+0 37 2.3180	.0423	-.002968	0.003943	0.19263	0.01636
537	0	18	11.30	5 20 54.91830	.01172	+0 44 22.3728	.0329	-.030249	0.003910	-0.27589	0.01467
538	0	17	11.15	5 21 2.30257	.01374	+0 17 25.1702	.0352	0.023786	0.005631	-0.93363	0.01917
539	0	21	11.05	5 21 5.38516	.01742	+0 12 31.1913	.0465	0.031307	0.006541	-0.00200	0.02130
540	0	30	10.93	5 21 11.54830	.00865	+0 1 27.9824	.0281	-.027969	0.003131	-0.43370	0.01490
541	48405	29	7.50	5 21 16.65837	.01655	+0 49 3.1370	.0817	-.009181	0.006097	0.10333	0.02836
542	0	17	11.15	5 21 18.84632	.01197	+0 30 17.1463	.0379	-.030380	0.004266	0.39372	0.01543
543	0	31	10.80	5 21 19.63549	.00772	+0 3 30.9811	.0330	-.121101	0.003267	-0.75220	0.01205
544	0	33	9.75	5 21 24.85203	.01323	+0 10 22.6152	.0421	0.006270	0.007586	-0.48527	0.02286
545	0	17	10.65	5 21 27.75414	.00835	+0 34 55.8892	.0379	0.006225	0.002946	-0.72837	0.01366
546	0	18	11.00	5 21 31.59654	.01355	+0 38 38.1299	.0287	-.053554	0.005301	-2.01525	0.00928
547	48442	42	8.50	5 21 34.39767	.00896	+0 8 49.5903	.0395	-.038030	0.006295	-0.92257	0.02220
548	0	31	10.90	5 21 39.64259	.00792	+0 0 11.2017	.0316	0.007078	0.003217	-0.82518	0.01480
549	0	21	9.50	5 21 45.15175	.01071	+0 14 57.3568	.0418	0.020537	0.003195	-0.81319	0.01570
550	0	15	9.25	5 21 49.00615	.01007	+0 50 1.5027	.0431	-.006507	0.003243	0.16251	0.01610

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
551	0	20	11.25	5 22 1.89584	.01292	+0 43 45.1679	.0446	-.024002	0.004379	-0.09584	0.01936
552	0	31	10.97	5 22 4.80327	.01049	+0 0 28.3493	.0334	-.050423	0.005481	-1.35604	0.01751
553	0	33	8.50	5 22 9.14908	.00758	+0 40 2.4832	.0385	-.086713	0.002582	0.76337	0.01433
554	0	17	10.25	5 22 11.40914	.00988	+0 25 16.4558	.0429	0.020727	0.003900	-0.76240	0.01654
555	48577	43	7.95	5 22 15.48297	.00681	+0 43 19.9344	.0253	-.002164	0.002943	-0.16470	0.01188
556	0	20	9.17	5 22 19.03845	.00879	+0 30 30.3885	.0356	0.048112	0.002881	-0.81651	0.01098
557	0	18	10.90	5 22 19.37278	.01384	+0 28 35.4412	.0462	-.024724	0.004268	0.57731	0.01996
558	0	28	10.50	5 22 22.25484	.00934	+0 10 33.6869	.0336	-.022844	0.004348	-0.73583	0.02241
559	48600	35	8.50	5 22 25.03999	.00777	+0 31 57.3964	.0271	-.005024	0.003083	-0.08712	0.01207
560	0	22	9.97	5 22 25.43689	.00877	+0 21 24.7844	.0300	0.019259	0.003405	-0.62400	0.01284
561	0	20	11.17	5 22 25.99927	.01605	+0 22 14.7706	.0510	-.013060	0.005785	0.07442	0.01828
562	0	32	10.67	5 22 26.78310	.00886	+0 9 27.2177	.0284	-.007087	0.004244	-0.51767	0.01424
563	0	18	10.87	5 22 28.10152	.01254	+0 6 50.5673	.0406	-.063362	0.006840	-1.49374	0.02272
564	0	32	11.03	5 22 29.64780	.01505	+0 11 28.1401	.0735	-.035645	0.008600	0.88978	0.03535
565	0	24	8.53	5 22 32.57910	.01224	+0 53 23.2816	.0433	-.030509	0.004236	0.22125	0.01551
566	0	17	10.70	5 22 40.91241	.00909	+0 54 12.2350	.0462	-.113801	0.002334	1.01023	0.01755
567	0	72	8.85	5 22 40.22639	.00729	+0 7 0.0872	.0249	-.005681	0.003059	-0.97502	0.01558
568	0	33	8.47	5 22 45.26297	.01266	+0 16 14.9674	.0616	-.111905	0.006402	-0.74790	0.02952
569	0	25	9.45	5 22 59.72662	.00863	+0 17 53.8719	.0443	0.022187	0.003372	-0.04206	0.01365
570	48696	50	8.20	5 23 1.95389	.00867	+0 19 11.5513	.0383	0.024079	0.004807	-0.62369	0.02056
571	0	24	10.80	5 23 4.92712	.01247	+0 18 50.9390	.0369	-.037194	0.007332	-0.49017	0.01153
572	0	44	9.45	5 23 9.95330	.00617	+0 40 46.8494	.0290	0.018315	0.002971	1.24925	0.01389
573	48725	30	5.60	5 23 12.69736	.01163	+0 28 38.3076	.0645	-.041785	0.005871	0.86506	0.02986
574	0	37	9.85	5 23 16.11112	.00738	+0 45 14.7429	.0304	-.024546	0.001615	-0.01839	0.01411
575	0	65	8.35	5 23 30.05681	.00754	+0 41 5.6284	.0258	-.004889	0.002975	0.09013	0.00934
576	0	52	9.00	5 23 30.03478	.00697	+0 38 28.3513	.0301	-.029286	0.002526	1.30112	0.01104
577	0	29	10.95	5 23 30.53863	.01514	+0 29 46.3818	.0277	-.004729	0.005858	0.24059	0.00870
578	0	18	11.30	5 23 30.97558	.01172	+0 51 31.3310	.0818	-.060449	0.004448	-2.02062	0.02927
579	48796	59	7.40	5 23 31.31151	.01204	+0 47 29.0287	.0428	-.011468	0.004528	0.45777	0.01559
580	48804	54	9.70	5 23 34.45862	.00787	+0 11 51.1174	.0288	-.063343	0.004555	-0.06429	0.01641
581	0	23	10.75	5 23 38.20855	.00737	+0 49 58.8648	.0380	0.075766	0.001172	-0.77047	0.01178
582	0	25	11.25	5 23 39.19796	.01355	+0 18 15.4598	.0546	-.075781	0.005794	-0.97048	0.02139
583	0	54	8.40	5 23 42.53131	.00867	+0 6 43.6791	.0332	-.028297	0.002431	-0.72629	0.01632

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
584	48841	42	8.85	5 23 45.40528	.00933	+0 45 24.6936	.0352	-.000324	0.004439	-0.38471	0.01435
585	48848	48	7.50	5 23 47.75946	.01205	+0 45 21.2470	.0525	-.019984	0.005578	-0.25758	0.02003
586	0	41	9.25	5 23 49.33350	.00912	+0 13 10.6639	.0470	0.095268	0.001519	-0.82284	0.02589
587	48860	35	8.90	5 23 52.50705	.00743	+0 47 49.5027	.0338	-.007324	0.003342	0.13950	0.01337
588	0	22	11.03	5 23 57.84823	.01900	+0 6 25.4357	.0428	-.039706	0.006307	-0.37517	0.01782
589	48876	47	8.65	5 23 58.58868	.00949	+0 29 6.4281	.0448	0.093485	0.003998	0.99620	0.01669
590	0	15	10.95	5 24 4.07131	.02170	+0 47 30.0095	.0870	0.008443	0.003620	-1.44401	0.03058
591	0	37	8.97	5 24 3.44149	.00909	+0 0 0.2594	.0356	-.033396	0.003640	-0.27977	0.01548
592	0	23	10.95	5 24 5.09524	.01200	+0 42 31.5633	.0357	0.002139	0.004666	-0.52963	0.01395
593	0	21	11.20	5 24 14.71770	.01964	+0 25 9.2411	.0641	0.251275	0.006914	-3.11185	0.02399
594	48954	42	7.95	5 24 20.66671	.01212	+0 36 6.4913	.0348	0.045335	0.005394	-1.10336	0.01318
595	0	7	11.20	5 24 23.35554	.01944	+0 55 12.8739	.1449	-.081135	0.004992	-4.66897	0.05276
596	0	43	8.90	5 24 29.54356	.00797	+0 41 44.4980	.0346	0.022256	0.002947	0.05734	0.01277
597	0	34	8.80	5 24 31.27116	.00518	+0 22 38.5674	.0379	-.077655	0.002310	0.08436	0.01402
598	0	28	11.15	5 24 32.47284	.01054	+0 12 33.8812	.0347	-.047281	0.005489	0.28971	0.02086
599	0	31	9.05	5 24 33.20035	.00805	+0 45 5.8522	.0302	-.025787	0.004019	-0.09158	0.01047
600	0	49	8.80	5 24 38.29067	.00724	+0 5 8.3535	.0276	-.022696	0.003052	-0.04030	0.01268
601	0	33	9.00	5 24 39.05824	.00595	+0 39 38.2683	.0290	0.036849	0.001634	-0.47095	0.01050
602	0	33	8.73	5 24 50.96233	.02873	+0 3 57.2541	.0830	-.073602	0.009054	-1.71971	0.02359
603	0	20	11.25	5 24 53.77764	.01262	+0 24 23.6740	.0330	-.001929	0.005128	-0.38885	0.01210
604	0	56	7.80	5 24 56.70776	.00727	+0 12 40.4129	.0262	0.060428	0.004083	-0.60828	0.01537
605	0	40	9.00	5 24 59.95824	.00704	+0 1 46.7287	.0321	-.065873	0.003261	-0.81009	0.01520
606	49076	56	7.80	5 25 1.33793	.01111	+0 39 39.6121	.0450	0.059836	0.004091	1.33077	0.01681
607	0	25	10.60	5 25 5.95978	.01852	+0 1 26.2185	.0428	-.092451	0.005734	0.01166	0.01899
608	0	27	9.40	5 25 6.54205	.00856	+0 22 47.4236	.0482	-.103167	0.003942	-1.21462	0.01816
609	0	26	11.10	5 25 11.07851	.01454	+0 22 1.6668	.0500	-.034342	0.005671	0.01375	0.01912
610	0	3	10.30	5 25 13.08120	.02610	+0 35 4.8842	.1701	0.157289	0.014655	8.05850	0.09779
611	0	34	10.60	5 25 18.49170	.01071	+0 43 4.7234	.0289	0.021980	0.004372	-0.01718	0.01006
612	0	13	10.00	5 25 19.46297	.05929	+0 36 36.4742	.1295	0.217443	0.021401	0.62063	0.03357
613	0	21	9.50	5 25 23.32039	.01918	+0 48 39.8535	.0705	-.052165	0.006430	1.28322	0.02591
614	0	30	11.05	5 25 25.66809	.00984	+0 24 37.4331	.0325	-.035525	0.004459	0.06775	0.01411
615	0	39	9.70	5 25 28.63411	.00936	+0 3 7.3909	.0396	-.025670	0.004401	-3.66854	0.02601
616	0	58	9.40	5 25 30.85891	.00616	+0 9 18.1342	.0256	-.054317	0.004249	-1.14916	0.02064

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
617	49203	99	7.05	5 25 35.48502	.00527	+0 44 49.6798	.0210	-.060216	0.002643	-0.57817	0.00936
618	0	33	10.90	5 25 43.87218	.00918	+0 28 25.9927	.0332	0.030491	0.003815	0.89881	0.01455
619	0	33	10.30	5 25 49.18903	.01077	+0 52 36.3817	.0234	0.161474	0.004176	-0.73331	0.00840
620	0	40	10.15	5 25 49.72249	.01154	+0 18 18.4282	.0304	-.033452	0.004344	-0.84176	0.01369
621	49259	79	6.00	5 25 52.02077	.00618	-0 1 9.8968	.0298	-.084807	0.003460	-1.62851	0.01484
622	0	99	7.90	5 25 59.53494	.00427	+0 43 34.9503	.0156	-.064084	0.002335	-0.06807	0.00716
623	0	30	11.05	5 26 3.35566	.01256	+0 26 28.2287	.0479	-.032642	0.004611	-0.12793	0.01771
624	0	38	11.30	5 26 16.31435	.00882	+0 40 39.4794	.0408	0.005726	0.004171	-0.03068	0.01612
625	0	32	11.45	5 26 18.80754	.00780	+0 47 23.1512	.0317	-.024936	0.001920	0.17972	0.01540
626	0	41	11.47	5 26 24.18562	.00736	+0 40 34.3745	.0261	0.031348	0.004450	0.95943	0.01395
627	0	32	7.70	5 26 26.66117	.01420	+0 16 5.0521	.0659	0.015797	0.004218	-0.10197	0.02433
628	49402	75	7.73	5 26 40.09933	.00718	+0 9 36.5020	.0271	0.246403	0.004104	-2.11465	0.01954
629	0	28	8.80	5 26 43.24169	.01134	+0 56 55.1670	.0437	-.040350	0.007949	0.59403	0.03678
630	0	38	11.35	5 26 42.89497	.01359	+0 40 38.4200	.0267	-.149551	0.007815	0.94367	0.00905
631	0	27	11.35	5 26 42.71586	.01450	+0 9 19.0366	.0580	-.039377	0.005584	0.03861	0.01848
632	0	37	9.70	5 26 55.51853	.00857	+0 47 6.8444	.0523	-.085384	0.003915	-4.85175	0.01943
633	0	23	10.45	5 27 2.32696	.01204	+0 16 38.6072	.0361	-.072563	0.003618	-0.24268	0.01025
634	0	23	11.00	5 27 7.81302	.01336	+0 25 57.8387	.0259	-.040486	0.006372	-1.52094	0.00804
635	0	21	10.80	5 27 8.23972	.00894	+0 22 38.0205	.0268	0.000189	0.002582	-1.14426	0.01322
636	49489	52	8.80	5 27 11.23807	.00577	+0 8 17.8464	.0319	-.110973	0.004347	-1.80685	0.02994
637	0	28	11.17	5 27 16.09472	.01131	+0 6 12.3894	.0319	-.030024	0.004373	-0.03632	0.01058
638	0	21	11.05	5 27 27.47690	.01027	+0 31 50.5852	.0235	-.032126	0.003381	-1.32662	0.01033
639	0	64	8.90	5 27 30.63231	.00500	+0 39 4.6383	.0167	-.036452	0.002060	-2.07861	0.00736
640	0	33	11.10	5 27 37.41037	.00678	+0 35 51.3530	.0241	-.053122	0.003200	-1.53692	0.01131
641	0	29	10.90	5 27 40.50762	.00595	+0 54 58.8420	.0248	-.178102	0.004178	0.00259	0.01098
642	0	19	10.45	5 27 42.82768	.01087	+0 16 22.2708	.0527	0.008593	0.003788	0.41326	0.02407
643	0	35	11.00	5 27 48.69342	.00782	+0 46 0.7559	.0238	-.050469	0.004013	-0.41089	0.01243
644	0	45	9.05	5 27 49.94811	.00684	+0 34 13.1702	.0254	-.050052	0.003813	-0.00401	0.01032
645	49642	79	8.75	5 27 55.18740	.00432	+0 44 43.9656	.0158	0.178935	0.002432	-1.17184	0.00917
646	49653	46	9.45	5 27 59.60583	.00743	+0 8 26.0308	.0306	-.016626	0.004357	-0.00417	0.01977
647	0	87	9.90	5 28 0.75016	.00405	+0 36 39.6485	.0130	-.074686	0.002266	0.24733	0.00722
648	49654	46	7.80	5 28 0.54920	.00638	+0 19 42.1407	.0475	-.067727	0.003034	-0.12489	0.01861
649	0	47	9.55	5 28 10.93481	.00776	+0 19 14.3174	.0452	0.057334	0.002796	-1.71366	0.01814

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
650	0	47	9.85	5 28 15.41350	.00546	+0 44 56.2295	.0199	-.026122	0.002537	-1.75526	0.01150
651	49698	94	7.63	5 28 14.90206	.00462	-0 0 30.2286	.0171	0.047634	0.002949	0.40785	0.00941
652	0	23	11.05	5 28 20.79780	.01077	+0 15 37.9144	.0360	-.008715	0.003807	-0.02851	0.01431
653	0	32	9.30	5 28 25.48665	.00698	+0 22 21.1063	.0490	-.012692	0.002462	0.66002	0.01850
654	0	22	10.95	5 28 29.62163	.01068	+0 29 48.0834	.0260	-.003882	0.004042	-1.72942	0.00670
655	0	42	9.85	5 28 30.21916	.00908	+0 39 17.7709	.0340	-.086954	0.003679	-0.08477	0.01447
656	0	37	9.85	5 28 30.47032	.00969	+0 29 0.5374	.0345	0.263659	0.003747	-1.96871	0.01316
657	0	40	10.90	5 28 35.38584	.00926	+0 35 8.8698	.0307	-.081770	0.007695	-0.55015	0.02119
658	0	25	10.40	5 28 38.15326	.00669	+0 49 34.0304	.0261	0.025258	0.003403	0.15295	0.01189
659	0	30	11.15	5 28 45.16194	.01053	+0 18 10.4510	.0534	0.002351	0.002435	-2.41393	0.02055
660	0	39	11.05	5 28 56.59323	.00911	+0 35 9.2604	.0213	-.067749	0.004232	1.16579	0.00907
661	0	46	10.35	5 29 2.00882	.00753	+0 40 4.5347	.0282	-.027754	0.003402	1.42432	0.01250
662	0	26	11.10	5 29 3.57873	.01215	+0 10 16.6472	.0478	0.032744	0.002890	-0.08036	0.02495
664	0	27	9.85	5 29 12.82021	.01582	+0 12 32.0503	.0459	-.016148	0.004211	-3.09411	0.01675
665	0	34	10.53	5 29 19.03523	.01010	-0 1 22.0594	.0375	-.059935	0.002747	0.36447	0.01484
666	0	34	11.27	5 29 29.23000	.01053	+0 2 22.5948	.0329	0.031368	0.004211	-1.44848	0.01234
667	0	38	10.10	5 29 34.86342	.00874	+0 5 30.1201	.0344	-.020057	0.003404	-0.26321	0.01539
668	0	25	10.95	5 29 35.99760	.00698	+0 51 28.2086	.0221	-.021931	0.003375	1.49268	0.01018
669	0	19	11.00	5 29 35.22417	.01644	+0 11 58.8967	.0518	-.008239	0.006900	0.48203	0.01105
670	0	42	9.65	5 29 36.44152	.00594	+0 51 34.1987	.0204	-.014732	0.002691	0.55059	0.00873
671	0	56	9.55	5 29 38.12589	.00583	+0 45 33.3684	.0128	-.043100	0.002587	1.05783	0.00717
672	0	25	11.15	5 29 39.69563	.00692	+0 52 19.9593	.0197	-.049778	0.003670	1.32348	0.00859
673	0	11	11.25	5 29 47.52400	.01589	+0 21 28.0195	.0374	-.000716	0.002836	-0.24439	0.00699
674	0	16	10.50	5 29 56.12037	.01017	+0 27 49.5271	.0426	-.068906	0.003000	-0.49043	0.01606
675	0	50	9.60	5 29 56.25565	.00817	+0 0 55.8140	.0319	0.056009	0.004612	0.31132	0.01705
676	50023	44	7.23	5 30 4.19919	.00683	-0 1 21.7658	.0317	0.020308	0.004602	-1.55926	0.01912
677	0	25	11.05	5 30 5.50663	.01549	+0 9 26.6750	.0500	0.033828	0.004587	-1.01258	0.03938
678	0	16	10.25	5 30 12.17181	.01406	+0 26 18.8077	.0311	0.168483	0.005205	2.13184	0.01429
679	0	27	10.60	5 30 17.76985	.01535	+0 9 42.5946	.0419	-.084247	0.006226	-0.01235	0.02246
680	0	18	10.23	5 30 23.44378	.01140	+0 13 34.2990	.0417	0.043907	0.002979	1.20408	0.01587
681	0	26	10.53	5 30 34.84494	.00735	+0 54 27.1002	.0193	0.023129	0.003526	2.39608	0.01039
682	0	49	9.07	5 30 36.92530	.00659	+0 50 1.4153	.0205	0.005986	0.003143	1.09422	0.00670
683	0	17	10.40	5 30 42.95774	.01316	+0 13 11.4586	.0435	0.118792	0.005386	2.78184	0.01500

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
684	0	18	10.95	5 30 46.26538	.01384	+0 19 51.3956	.0481	-.026683	0.003849	1.82087	0.02016
685	50177	13	7.70	5 30 51.57245	.02386	+0 35 15.0276	.0897	-.098693	0.004786	-0.79302	0.04593
695	0	36	10.25	5 30 32.28025	.00911	-0 2 5.3430	.0306	-.000785	0.003778	0.49891	0.01348
696	0	17	10.90	5 31 13.03612	.01471	+0 53 36.1253	.0389	-.013284	0.011561	-0.59950	0.03085
697	0	20	10.90	5 31 15.01750	.01006	+0 4 38.8357	.0509	0.009407	0.002773	0.81546	0.02042
698	0	17	11.05	5 31 15.62488	.01035	+0 15 2.5487	.0473	0.059668	0.003395	0.50342	0.01936
699	0	15	10.15	5 31 19.67425	.01718	+0 44 6.5807	.0469	-.093569	0.009607	-0.26226	0.03373
700	0	17	9.95	5 31 20.80599	.01422	+0 19 8.3212	.0429	0.043338	0.003221	1.54997	0.01539
701	0	8	10.20	5 31 21.24113	.06661	+0 21 1.2664	.1234	-.009106	0.012858	0.79734	0.03883
702	0	13	9.50	5 31 27.51231	.01977	+0 22 43.4914	.0835	-.037895	0.005886	-0.56758	0.05116
703	0	30	11.25	5 31 37.05174	.01599	+0 5 48.8207	.0509	-.030128	0.008894	0.59303	0.02043
704	0	27	9.05	5 31 37.81238	.01120	+0 50 20.8170	.0358	-.059763	0.007285	-0.92389	0.00788
705	0	13	8.40	5 31 38.15886	.01732	+0 22 51.6643	.0602	-.094743	0.004669	0.28017	0.01924
706	0	23	8.95	5 31 47.34805	.01410	+0 38 59.1947	.0505	-.098758	0.005082	-0.83962	0.03728
707	0	17	10.95	5 31 52.46131	.01507	+0 53 37.5513	.0541	-.048665	0.009022	-1.16209	0.01837
708	0	17	9.85	5 31 57.17702	.03254	+0 44 6.2448	.1120	-.198683	0.014891	-0.26855	0.03329
709	0	20	9.65	5 32 3.08343	.01480	+0 43 3.8116	.0684	-.083216	0.011469	-0.57622	0.05263
710	0	17	11.35	5 32 11.48984	.01678	+0 50 58.6659	.0615	-.038849	0.007528	-1.00043	0.04445
711	0	8	9.10	5 32 16.51526	.03171	+0 37 15.4990	.0759	-.091449	0.006562	-0.66647	0.02079
712	0	36	8.20	5 32 19.36187	.01477	+0 55 21.9592	.0343	-.204608	0.012474	0.46211	0.02814
713	0	18	9.95	5 32 19.15928	.01766	+0 8 17.7916	.0583	0.078411	0.004638	2.34938	0.02113
714	0	10	10.35	5 32 21.36507	.01950	+0 17 36.0711	.0465	0.010615	0.006084	0.97933	0.00970
715	50459	56	8.43	5 32 21.51043	.00850	+0 4 11.6588	.0469	-.001756	0.002563	1.43945	0.01741
716	0	27	10.83	5 32 27.24808	.01565	+0 3 25.1330	.0784	0.016086	0.005491	0.11404	0.02580
717	0	11	10.95	5 32 32.54334	.02322	+0 54 33.7486	.0587	-.051213	0.011159	-1.24992	0.01307
718	0	29	10.37	5 32 35.10747	.01172	+0 4 20.8920	.0451	-.055262	0.003593	1.22978	0.01497
722	0	14	9.40	5 32 44.72079	.01796	+0 13 4.7889	.0485	-.003347	0.005113	1.39753	0.01116
725	0	17	9.30	5 32 49.46616	.01215	+0 12 29.5325	.0853	0.006469	0.002849	0.67291	0.02466
728	50624	52	8.07	5 33 1.06804	.01252	+0 4 4.4714	.0600	0.027290	0.004745	1.95542	0.02018
734	0	19	11.10	5 33 50.17199	.01877	+0 5 24.6302	.0645	0.014411	0.005180	0.99620	0.01670
735	0	18	11.45	5 33 54.75082	.01927	+0 2 57.9144	.0711	-.018848	0.005851	1.25709	0.01923
736	0	13	10.57	5 33 55.48018	.02386	+0 1 29.3495	.0692	-.237389	0.007368	3.16659	0.02311
738	0	5	9.75	5 34 3.29764	.04609	+0 14 39.6294	.2314	-.059213	0.011491	2.74217	0.03947

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
739	0	13	11.15	5 34 5.63528	.02057	+0 6 35.6118	.1055	-.022174	0.005258	2.00042	0.03344
742	0	20	10.40	5 34 21.91572	.01630	+0 0 31.8573	.0520	-.013113	0.004541	1.33244	0.01535
743	0	39	8.20	5 34 25.95937	.01068	+0 4 15.2624	.0359	-.033055	0.003617	1.87701	0.00957
745	0	21	10.88	5 34 27.28478	.01887	+0 3 54.8414	.0622	-.005876	0.005780	1.30888	0.01404
746	0	14	11.50	5 34 27.64516	.02319	+0 3 59.5933	.0890	0.048758	0.008975	0.56105	0.01824
811	0	9	10.70	5 13 56.25840	.04855	-0 55 13.6513	.0805	-.008539	0.010378	-0.20838	0.01672
812	47131	23	8.55	5 14 2.07161	.01823	-0 54 20.3157	.0572	0.013078	0.008087	-0.55330	0.02020
814	47163	29	7.87	5 14 11.14562	.01941	-0 56 32.9337	.0586	-.074633	0.007049	0.48355	0.01758
831	0	12	10.63	5 14 59.54874	.02718	-0 58 22.4924	.0845	-.151474	0.005706	1.79195	0.02203
832	47341	7	8.35	5 15 10.51232	.01955	-0 25 21.4129	.0769	-.075257	0.005418	0.18388	0.02562
833	0	3	11.20	5 15 11.55116	.10227	-0 40 28.8000	.2729	-.060228	0.015676	-0.30238	0.03913
834	0	13	10.87	5 15 19.22314	.02714	-0 58 45.6129	.0630	-.040523	0.004392	-0.54089	0.01721
835	47361	14	8.35	5 15 21.67112	.02658	-0 20 10.5326	.0825	-.053337	0.005952	0.43938	0.02078
836	0	5	11.45	5 15 24.49832	.03932	-0 34 32.1345	.0856	-.057248	0.010114	-0.35143	0.01461
840	0	5	11.15	5 15 36.76221	.03484	-0 20 47.0221	.0564	-.041234	0.006777	1.23182	0.01040
843	0	6	10.50	5 15 40.18036	.03419	-0 27 3.0660	.0549	-.059823	0.009178	0.08146	0.01032
844	0	19	10.35	5 15 41.78909	.01709	-0 47 53.1180	.0330	-.026660	0.004341	0.28992	0.00939
845	0	38	9.70	5 15 46.86695	.01067	-0 54 11.7129	.0533	-.025141	0.003288	-0.01561	0.01822
846	0	18	10.10	5 15 47.32146	.01749	-0 39 52.1481	.0897	-.018269	0.005832	-0.07222	0.03175
847	0	19	10.15	5 15 47.56872	.01201	-0 42 26.4675	.0401	0.019989	0.003056	0.12974	0.01166
848	0	3	11.10	5 15 55.84411	.03114	-0 30 8.7971	.1653	-.080130	0.004770	0.02114	0.02832
849	0	6	10.45	5 16 2.22209	.02411	-0 26 11.2133	.0812	-.057805	0.005123	-0.21459	0.02535
850	0	6	10.70	5 16 3.14095	.02801	-0 20 29.5056	.0692	-.087512	0.008601	-0.42384	0.01498
851	0	21	10.90	5 16 3.23291	.01161	-0 48 3.3701	.0691	-.066081	0.003040	0.27688	0.02413
852	0	18	11.05	5 16 11.79340	.01323	-0 48 34.6308	.0464	-.013715	0.002535	1.90515	0.01591
853	0	6	10.35	5 16 15.86618	.01843	-0 30 39.6230	.0508	-.058357	0.002607	0.68899	0.01159
854	0	3	11.25	5 16 20.25603	.03868	-0 21 29.6661	.1400	-.075882	0.007517	-0.97632	0.02077
855	0	14	10.75	5 16 24.61082	.02670	-0 43 20.8911	.0527	0.061164	0.004907	-0.82507	0.01361
856	0	18	11.07	5 16 27.77425	.02080	-0 55 3.6188	.0600	-.044472	0.005641	0.69584	0.01569
859	0	31	9.90	5 16 44.34013	.01847	-0 40 9.0991	.0835	-.009894	0.006302	-1.04903	0.02884
861	0	14	11.10	5 16 56.10704	.01686	-0 41 42.4677	.0794	-.067772	0.004285	-0.12913	0.02591
862	0	6	10.80	5 16 56.78904	.02275	-0 30 13.6363	.0807	-.038975	0.005848	1.07420	0.02435
863	514045	15	9.05	5 17 0.17494	.02832	-0 38 51.5650	.0992	-.067877	0.010263	-0.24976	0.03589

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
864	0	19	10.25	5 17 0.41877	.01350	-0 42 30.4342	.0665	-.049816	0.003815	-0.19241	0.02240
865	0	4	10.75	5 17 16.09590	.03099	-0 17 8.9289	.0970	-.079237	0.006502	0.10655	0.02429
866	0	6	10.35	5 17 19.17397	.02254	-0 32 21.0864	.0449	-.063262	0.007405	-0.55227	0.01028
867	0	6	10.80	5 17 19.49860	.01995	-0 25 0.0235	.0607	-.060561	0.004456	-0.11576	0.01796
868	0	8	9.65	5 17 20.48687	.02575	-0 20 46.4457	.0709	-.094237	0.005821	-0.20758	0.02526
869	0	12	11.10	5 17 28.91404	.01649	-0 40 5.7567	.0749	-.107820	0.002846	0.75124	0.01998
871	0	3	10.95	5 17 37.01862	.03153	-0 17 16.8931	.1715	-.066352	0.006753	-0.45594	0.03471
872	0	6	10.55	5 17 38.20650	.01750	-0 19 5.2414	.0773	-.022081	0.003636	1.15745	0.02252
873	0	6	11.15	5 17 48.18214	.02050	-0 28 43.6785	.0676	-.031200	0.004187	0.08061	0.02275
875	0	14	10.67	5 17 58.02858	.01996	-0 55 23.0965	.0587	-.127041	0.005606	-2.89348	0.01609
876	0	10	10.60	5 18 0.26619	.04614	-0 44 59.5401	.2052	0.212052	0.006879	-2.13115	0.03538
877	0	6	10.15	5 18 16.92422	.01943	-0 21 29.0705	.0731	-.080360	0.005830	-0.79969	0.02230
879	0	6	10.95	5 18 19.99266	.02200	-0 29 25.5738	.0590	-.081138	0.005923	-0.87167	0.00900
881	47982	28	8.00	5 18 47.55807	.02583	-0 39 21.8932	.0795	-.068286	0.008052	-0.59803	0.02552
882	0	17	11.40	5 26 22.42113	.01593	+0 27 22.0916	.0556	-.052253	0.006479	0.12525	0.02390
885	48012	15	4.90	5 18 58.64613	.03700	-0 27 52.0559	.1034	-.025992	0.011429	0.46291	0.03262
886	0	34	9.70	5 19 12.41820	.01108	-0 19 58.5411	.0308	-.115161	0.004457	0.32241	0.01132
887	0	20	3.85	5 19 12.47982	.03633	-0 25 48.9262	.1448	-.027120	0.012631	-0.10669	0.05172
888	0	38	9.00	5 19 38.49256	.00828	-0 9 36.8572	.0286	-.073829	0.005254	-0.14550	0.00503
889	0	32	10.55	5 19 46.15765	.01215	-0 16 22.1899	.0276	-.012325	0.004981	0.70725	0.01114
890	0	20	11.15	5 19 58.63135	.01866	-0 27 21.8063	.0565	-.061302	0.006295	-0.14183	0.01912
891	0	22	10.95	5 20 6.36294	.00909	-0 28 39.4211	.0771	-.024813	0.001406	-0.13287	0.02685
892	0	31	10.45	5 20 11.69144	.00811	-0 48 17.1064	.0325	-.054012	0.003583	-1.21754	0.01342
893	0	30	10.50	5 20 15.75008	.00842	-0 5 9.7199	.0227	-.042338	0.002927	-0.49780	0.00711
894	0	65	9.15	5 20 21.31618	.00499	-0 2 36.4369	.0171	0.032534	0.000965	-0.57915	0.00863
895	0	28	10.10	5 20 48.27221	.00966	-0 7 6.1679	.0284	-.031019	0.001477	0.15072	0.00798
896	0	53	8.75	5 20 59.56977	.00602	-0 6 40.7549	.0222	-.023147	0.002060	0.17636	0.01097
897	48371	60	8.07	5 21 7.48427	.00560	-0 54 28.0750	.0244	-.018145	0.004535	-0.85453	0.01476
898	48378	25	4.65	5 21 8.75411	.01930	-0 12 18.9126	.0743	-.046712	0.007632	-1.37195	0.02844
899	0	20	11.15	5 21 10.17068	.01369	-0 11 44.5921	.0380	0.045194	0.003769	2.32103	0.00786
900	48392	74	9.20	5 21 13.60591	.00447	-0 59 27.6490	.0172	-.060586	0.003357	0.21199	0.01025
901	0	22	11.00	5 21 17.48398	.01120	-0 2 31.4726	.0283	-.022753	0.005449	-0.41263	0.01853
902	48404	35	6.20	5 21 18.66210	.01441	-0 54 44.2403	.0668	-.079997	0.005528	1.27101	0.02333

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
903	0	36	11.00	5 21 19.14552	.00792	-0 51 1.4016	.0262	-.087824	0.004398	-0.96941	0.01733
904	0	21	11.10	5 21 20.76862	.01143	-0 6 29.6520	.0325	-.035599	0.003244	-0.29117	0.01696
905	0	3	9.05	5 21 22.31476	.04334	-0 7 18.3066	.1044	-.395603	0.010740	5.63350	0.01609
906	0	18	11.05	5 21 27.41092	.01691	-0 29 12.4218	.0532	-.146096	0.004955	-0.95889	0.01814
907	0	22	9.55	5 21 31.72463	.01459	-0 36 4.8487	.0599	-.126016	0.005061	0.82939	0.02169
908	0	20	11.30	5 21 34.82093	.01369	+0 1 15.7219	.0443	-.033748	0.004327	0.07641	0.03217
909	0	49	9.55	5 21 34.37682	.00712	-0 22 59.9024	.0311	-.108744	0.003048	-0.56995	0.01183
910	0	19	8.95	5 21 34.76291	.03993	-0 9 36.3777	.1288	0.077370	0.007425	-3.09460	0.03087
911	0	13	11.20	5 21 37.68981	.01503	-0 38 14.7093	.1023	-.072049	0.004710	-0.48673	0.03733
912	0	60	9.65	5 21 40.32149	.00940	-0 17 33.5567	.0255	-.108010	0.003829	-1.27351	0.01030
913	0	39	11.15	5 21 43.13484	.00919	-0 15 50.3270	.0285	-.110749	0.003704	-0.98294	0.01187
914	0	23	11.30	5 21 48.28184	.01469	-0 19 45.9491	.0390	0.085778	0.006065	-1.23530	0.01053
915	0	25	11.20	5 21 50.92553	.01741	-0 14 25.3512	.0482	-.070288	0.005915	-1.38390	0.00823
916	0	57	8.95	5 21 55.62335	.00599	-0 3 55.0305	.0177	-.052902	0.003213	-0.47384	0.01068
917	48506	30	6.07	5 21 56.23640	.01856	-0 56 13.7735	.0932	-.080091	0.006004	13.10583	0.03428
918	0	13	10.10	5 22 3.55552	.02371	-0 36 39.0146	.0198	-.029217	0.008533	-0.45614	0.00460
919	0	32	11.00	5 22 7.78717	.00882	-0 57 4.8250	.0291	-.000298	0.004604	0.48957	0.01377
920	48561	119	8.20	5 22 12.40997	.00423	-0 52 48.5428	.0205	-.156195	0.001949	-1.68461	0.00865
921	0	33	10.10	5 22 19.95625	.01021	-0 23 31.4241	.0299	-.078425	0.004047	-1.94025	0.01302
922	0	29	11.07	5 22 27.25190	.00940	-0 23 49.3452	.0419	-.029677	0.002734	-1.21540	0.01704
923	0	36	9.90	5 22 29.54517	.01378	-0 45 2.2452	.0525	0.036996	0.004492	-3.21472	0.01668
924	0	45	8.67	5 22 31.95155	.00702	-0 39 43.0808	.0686	-.039110	0.001973	-0.87009	0.02485
925	0	23	10.13	5 22 32.71811	.01271	-0 25 15.2517	.0547	-.083037	0.003714	-0.43415	0.01892
926	0	18	10.57	5 22 35.39455	.03710	-0 7 23.5838	.1118	0.052256	0.009588	-4.00153	0.03772
927	0	38	10.73	5 22 36.62959	.01530	-0 4 55.4749	.0323	-.053230	0.005557	-1.20419	0.01733
928	0	11	10.37	5 22 39.64479	.01073	-0 33 45.1176	.1041	0.084279	0.002886	-1.29836	0.02318
930	0	8	10.60	5 22 53.76951	.03435	-0 28 33.9542	.1175	-.053451	0.005430	-0.48332	0.03470
931	0	7	8.50	5 22 58.10084	.03530	-0 23 45.9830	.0861	-.041247	0.006150	-0.31128	0.02652
933	0	4	10.00	5 23 2.43426	.03808	-0 32 10.2561	.0898	-.032986	0.007665	-0.72001	0.01349
934	0	7	10.10	5 23 5.87511	.02518	-0 31 45.8835	.0719	-.117098	0.003937	-0.58921	0.02238
935	0	22	11.05	5 23 13.58298	.01770	-0 21 21.6192	.0417	-.058563	0.006420	-0.54193	0.01721
936	0	28	10.80	5 23 15.35619	.01202	-0 15 23.6968	.0455	-.063537	0.003283	0.73125	0.01738
937	0	32	10.15	5 23 16.15528	.00891	-0 51 10.9330	.0483	-.034972	0.002257	0.23313	0.02098

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
938	0	14	10.20	5 23 18.71118	.02115	-0 15 33.6418	.0564	0.005092	0.006211	3.70996	0.01673
939	48778	73	8.20	5 23 29.34691	.00630	-0 48 6.8671	.0216	-.062267	0.002747	-2.91313	0.00914
940	0	42	9.20	5 23 32.12591	.01012	-0 6 1.0169	.0373	-.045383	0.003091	-0.82638	0.01772
941	0	10	10.60	5 23 33.16494	.01674	-0 31 32.6691	.0517	-.032378	0.004318	-1.10992	0.01637
942	0	25	10.05	5 23 35.83513	.00891	-0 47 27.1446	.0499	0.035249	0.003373	-0.68968	0.01915
943	0	25	10.63	5 23 36.18473	.01212	-0 57 35.3717	.0352	-.064005	0.005418	-0.89475	0.02081
944	0	28	10.65	5 23 39.38669	.01237	-0 19 26.2739	.0386	-.059839	0.003339	-1.44528	0.01331
946	0	16	9.05	5 23 45.91354	.01709	-0 34 6.5895	.0390	-.009314	0.006111	-1.29149	0.01338
947	0	33	9.05	5 23 46.49915	.02052	-0 10 22.4054	.0556	-.125742	0.006413	0.49896	0.02057
948	0	21	10.40	5 23 53.94773	.02679	-0 3 23.0611	.0680	0.037697	0.006383	1.69284	0.02343
949	0	31	10.90	5 23 54.38488	.01053	-0 9 49.7858	.0328	-.037613	0.003036	-0.23922	0.00773
950	0	28	10.90	5 24 1.99139	.01297	-0 0 53.9682	.0352	-.046068	0.004697	-0.43669	0.01456
951	0	23	11.10	5 24 2.12442	.01241	-0 17 23.5647	.0484	-.069193	0.001947	0.02679	0.01758
952	0	31	10.35	5 24 2.03620	.00653	-0 49 28.7644	.0350	-.099389	0.002740	-0.06312	0.01162
953	514355	41	8.10	5 24 14.81826	.01374	-0 41 18.0346	.0919	-.028924	0.004778	-0.72312	0.03369
954	0	19	11.05	5 24 16.95601	.02203	-0 6 8.6363	.0458	-.056943	0.006552	0.22954	0.02728
955	0	9	11.15	5 24 18.11093	.01314	-0 34 11.9899	.1070	0.001009	0.004155	-2.34512	0.03826
956	0	65	8.60	5 24 19.56755	.00645	-0 12 15.8372	.0179	-.046898	0.002878	-0.33879	0.00776
957	48966	64	8.65	5 24 25.90198	.00743	-0 5 54.3802	.0228	-.051961	0.002952	-0.62347	0.01092
958	0	19	10.35	5 24 25.62840	.01397	-0 24 49.1413	.0423	-.090290	0.004541	-2.05628	0.01466
959	48979	44	8.55	5 24 31.70403	.02827	-0 24 44.8583	.0441	-.031886	0.010281	-2.08532	0.01520
960	0	28	10.60	5 24 33.14833	.01216	-0 13 25.5959	.0301	-.084844	0.005125	-1.82852	0.01415
961	0	20	11.15	5 24 35.96151	.01999	-0 12 28.4001	.0512	-.206734	0.008252	-1.56301	0.02234
962	0	27	10.95	5 24 37.32766	.01206	-0 14 41.1750	.0312	-.077938	0.003322	0.13586	0.01588
963	0	29	10.80	5 24 39.20666	.01252	-0 16 44.2522	.0338	-.092141	0.004975	-0.37616	0.01318
964	0	69	8.55	5 24 41.54915	.00737	-0 5 16.0612	.0241	0.013981	0.004051	0.23208	0.00616
965	0	29	10.30	5 24 41.96170	.01145	-0 20 22.3624	.0278	-.078772	0.004288	-0.16456	0.01157
966	0	26	10.10	5 24 49.46627	.01118	-0 22 35.8250	.0392	-.094048	0.003998	-0.97252	0.01629
967	0	29	10.95	5 24 58.93131	.01199	-0 5 13.3312	.0316	-.096596	0.005627	-0.44626	0.01618
968	0	31	10.60	5 25 1.75989	.01111	-0 8 58.5572	.0460	-.040006	0.003696	-0.12860	0.01734
969	514388	19	10.25	5 25 1.74599	.02822	-0 40 17.1033	.0693	-.023298	0.009137	-3.35143	0.02236
970	49074	47	8.05	5 25 2.40771	.01791	-0 40 24.9451	.1180	-.127655	0.006425	0.94395	0.04361
971	0	26	11.00	5 25 4.60878	.01096	-0 15 7.5741	.0377	-.068264	0.003447	-0.55644	0.00857

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon_{\mu\alpha}$	μ_δ	$\epsilon_{\mu\delta}$
972	0	21	10.55	5 25 5.94265	.01893	-0 44 21.8065	.0556	-.050843	0.006383	0.21039	0.01887
973	49103	52	8.75	5 25 7.37303	.01010	-0 18 34.9818	.0310	-.111634	0.004318	-0.08896	0.01270
974	49102	38	8.60	5 25 7.15733	.02690	-0 26 30.8893	.0963	-.154967	0.009894	-1.53233	0.03549
975	49105	66	8.30	5 25 7.83488	.00786	-0 1 36.8127	.0365	-.067352	0.004102	-1.51653	0.01592
976	49112	80	7.00	5 25 9.81147	.00839	-0 17 59.2541	.0386	-.088079	0.003411	-0.89714	0.01512
977	0	19	10.90	5 25 13.61445	.02167	-0 28 8.8659	.0323	-.069686	0.007392	-1.02907	0.00763
978	49164	81	7.90	5 25 24.89072	.00397	-0 59 56.1768	.0234	-.094028	0.002383	1.16749	0.00997
979	0	49	10.05	5 25 33.57485	.00669	-0 6 5.1244	.0221	-.012728	0.003514	-0.75410	0.01343
980	0	61	8.90	5 25 35.04389	.00698	-0 29 42.3397	.0200	-.012232	0.001848	0.15609	0.00934
981	0	58	9.70	5 25 48.12436	.00582	-0 45 46.7532	.0286	-.066841	0.001478	1.36203	0.01251
982	49254	90	7.35	5 25 51.89305	.00351	-0 44 31.7088	.0168	-.093651	0.001991	0.26569	0.00822
983	0	33	10.65	5 25 56.35108	.00747	-0 33 45.7168	.0264	-.295611	0.003338	1.13693	0.01282
985	0	51	10.80	5 26 0.26909	.00899	-0 22 8.9287	.0198	-.015170	0.004161	0.26000	0.01001
986	49305	95	7.55	5 26 8.38490	.00586	-0 30 3.4477	.0245	-.222838	0.002669	0.36823	0.01014
987	49336	92	6.70	5 26 19.56826	.00604	-0 38 32.4362	.0278	-.092022	0.002994	-0.04569	0.01133
988	49371	93	8.47	5 26 30.33126	.00486	-0 42 46.5582	.0182	-.060521	0.002161	0.20285	0.00830
989	0	65	9.93	5 26 36.37036	.00571	-0 45 12.0618	.0267	0.012153	0.003876	-4.00799	0.01546
990	49403	90	7.90	5 26 41.41520	.00484	-0 12 30.7020	.0183	0.020297	0.003704	0.82137	0.01226
991	49437	93	7.17	5 26 53.67859	.00581	-0 4 50.4700	.0319	0.036832	0.003171	-1.73338	0.01330
992	0	33	11.15	5 27 52.76305	.00788	+0 4 9.4320	.0272	0.097672	0.002172	-2.79433	0.01320
993	49874	73	8.30	5 29 10.08617	.00603	+0 4 2.4575	.0260	1.130133	0.003933	-46.21788	0.01677
994	0	4	10.95	5 26 35.99953	.13955	-0 22 44.5933	.3818	-.058686	0.019869	0.52925	0.05494
995	0	23	11.00	5 26 52.09387	.01039	-0 32 32.3701	.0270	0.003160	0.004503	0.57085	0.01223
996	0	29	10.75	5 26 57.13175	.00860	-0 23 43.2688	.0336	0.066870	0.001618	-1.43658	0.00747
997	0	28	10.65	5 26 57.14114	.01215	-0 23 2.8133	.0444	-.007869	0.004342	-1.44085	0.01505
998	0	25	10.80	5 26 58.28045	.00741	-0 29 46.5474	.0279	-.003679	0.003604	-0.04029	0.01296
999	0	51	9.70	5 27 0.35833	.00895	-0 41 16.4352	.0267	0.054344	0.004072	-0.14955	0.01249
1000	0	36	11.05	5 27 3.63895	.00763	-0 19 38.0904	.0298	-.003736	0.005612	-0.10698	0.01952
1001	49462	67	6.70	5 27 3.86540	.00928	-0 3 34.2442	.0452	-.040830	0.004457	-2.32020	0.01958
1002	49476	56	6.85	5 27 8.70301	.01099	-0 50 24.8857	.0545	0.374740	0.004903	-6.36631	0.02193
1003	514503	37	10.40	5 27 14.77524	.01018	-0 26 0.8158	.0333	-.026096	0.004855	0.34433	0.01354
1004	0	32	9.37	5 27 16.08478	.00951	-0 59 46.5241	.0329	0.002302	0.002764	-2.09357	0.01031
1005	0	38	10.90	5 27 18.25420	.00811	-0 27 3.1772	.0321	-.038543	0.004245	0.21442	0.01367

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon_{\mu\alpha}$	μ_δ	$\epsilon_{\mu\delta}$
1006	0	39	10.60	5 27 19.96401	.00605	-0 11 46.7773	.0259	-.047498	0.002560	-1.62680	0.01109
1007	0	31	11.25	5 27 24.72774	.00875	-0 9 6.2120	.0291	-.038778	0.004879	-1.03437	0.01381
1008	49530	133	9.45	5 27 25.14503	.00355	-0 18 17.7023	.0134	0.017985	0.002060	-0.07807	0.00748
1009	0	43	10.15	5 27 26.67231	.00981	-0 28 35.2414	.0346	0.164195	0.003542	-0.67341	0.01428
1010	0	32	11.15	5 27 29.69816	.00735	-0 4 48.3346	.0324	0.017749	0.003327	0.65092	0.01398
1011	0	37	11.00	5 27 31.92667	.00842	-0 28 13.0078	.0260	-.031816	0.002140	-0.44232	0.01715
1012	0	25	11.25	5 27 32.02379	.01813	-0 48 33.7925	.0590	-.037085	0.006741	0.19487	0.02290
1013	0	39	10.90	5 27 32.87499	.00689	-0 9 36.5869	.0180	-.004254	0.003212	-0.20501	0.00369
1014	0	37	11.20	5 27 38.83979	.00922	-0 30 50.5537	.0323	0.019091	0.004849	0.06068	0.01738
1015	0	33	10.35	5 27 42.78212	.00729	-0 5 18.9900	.0206	-.012196	0.004621	0.02070	0.00489
1016	49591	45	8.30	5 27 43.81750	.01619	-0 55 44.6620	.0646	-.031028	0.005810	0.19409	0.02340
1017	0	93	8.70	5 27 48.05094	.00405	-0 27 28.2246	.0146	0.017981	0.002342	0.47156	0.00725
1018	49641	122	8.55	5 27 56.14422	.00287	-0 21 36.7247	.0123	0.123377	0.001770	1.90713	0.00609
1019	49639	82	8.30	5 27 56.39632	.00411	-0 53 5.0661	.0173	0.035839	0.002168	-0.16096	0.00792
1020	0	23	10.63	5 28 5.60961	.01593	-0 54 22.1237	.0533	0.076123	0.005524	-1.69220	0.01863
1021	0	60	9.45	5 28 10.17542	.00914	-0 9 35.9394	.0342	0.027235	0.003181	2.37671	0.01850
1022	0	35	9.50	5 28 10.58390	.01283	-0 59 11.7080	.0384	-.011401	0.004223	-1.15567	0.01284
1023	49684	116	7.65	5 28 11.93043	.00613	-0 24 37.1673	.0241	0.004854	0.002646	0.06233	0.01021
1024	0	33	10.75	5 28 12.63890	.00637	-0 2 21.0835	.0198	0.007525	0.002661	3.38670	0.01103
1025	0	27	11.00	5 28 14.40212	.00986	-0 52 35.0731	.0341	0.032217	0.004318	0.63895	0.01515
1026	0	78	9.25	5 28 17.93884	.00461	-0 10 21.8973	.0171	0.022814	0.002558	0.21624	0.00889
1027	0	37	10.35	5 28 23.62940	.00793	-0 43 8.6895	.0260	0.034821	0.003619	0.32249	0.01178
1028	0	51	10.95	5 28 29.77107	.00827	-0 5 43.9918	.0264	0.028170	0.003412	-0.24892	0.01328
1029	0	34	10.23	5 28 31.11250	.00932	-0 56 6.8565	.0271	0.032967	0.003559	-0.18633	0.01072
1030	0	45	11.00	5 28 44.01835	.00867	-0 31 39.1397	.0342	-.009143	0.002987	-0.14455	0.01437
1031	0	109	9.05	5 28 45.46343	.00379	-0 13 24.0304	.0161	0.032929	0.001988	0.98541	0.00665
1032	49782	118	8.40	5 28 45.58817	.00448	-0 31 34.7805	.0145	0.054175	0.002840	0.63846	0.00829
1033	0	56	9.93	5 28 53.74324	.00595	-1 0 42.8243	.0170	0.251846	0.002938	-1.75815	0.00996
1034	0	49	10.75	5 28 57.54922	.00818	-0 30 42.7322	.0310	0.025779	0.003294	0.23945	0.01361
1035	0	55	10.70	5 29 0.93043	.00957	-0 27 38.5256	.0299	-.001660	0.003685	0.61468	0.01206
1036	0	38	11.05	5 29 5.85893	.01262	-0 6 11.5432	.0356	0.000796	0.005738	0.24100	0.01530
1037	0	35	10.95	5 29 9.00306	.01419	-0 2 18.8445	.0294	0.012101	0.004923	0.18698	0.00906
1038	0	33	10.90	5 29 12.76951	.01261	-0 6 32.1010	.0290	-.016394	0.003768	0.19378	0.00965

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
1039	0	24	11.35	5 29 15.04899	.01521	-0 3 23.1851	.0495	-.050217	0.002135	2.73140	0.02894
1040	0	33	10.80	5 29 15.64264	.01096	-0 4 42.5523	.0597	-.036733	0.002445	0.22575	0.02178
1041	0	23	11.10	5 29 16.54216	.01449	-0 45 36.3773	.0457	-.027513	0.003761	0.36770	0.02290
1042	0	15	9.80	5 29 21.33306	.01181	-0 51 50.3899	.0718	-.189303	0.007455	2.60353	0.02540
1043	0	31	10.65	5 29 21.84194	.01627	-0 35 17.3369	.0386	0.012138	0.005161	0.22471	0.01706
1044	0	48	7.15	5 29 26.98818	.01418	-0 19 11.6869	.0477	-.067441	0.004808	-0.92494	0.01639
1046	0	29	10.85	5 29 29.96025	.01283	-0 37 24.6951	.0303	-.014417	0.004308	0.60811	0.01150
1047	49928	99	7.65	5 29 30.76043	.00576	-0 27 52.0648	.0150	-.033910	0.002635	0.86163	0.00673
1048	0	31	10.95	5 29 34.18216	.01529	-0 10 10.2499	.0408	-.000244	0.005092	0.24399	0.01361
1050	0	71	9.70	5 29 37.57339	.00618	-0 41 20.4187	.0197	0.082617	0.002968	-0.66301	0.01092
1051	0	49	9.87	5 29 42.07811	.01004	-0 53 29.3581	.0300	-.047547	0.003824	0.14566	0.01680
1052	0	24	11.13	5 29 42.08485	.01601	-0 53 21.5435	.0454	-.000277	0.005412	-0.20579	0.01693
1053	0	45	10.45	5 29 43.56082	.00872	-0 19 38.8714	.0329	-.006980	0.003352	0.07296	0.01206
1054	0	43	11.35	5 29 48.35324	.01124	-0 19 46.6025	.0286	-.027347	0.004429	0.44940	0.01037
1055	0	32	10.70	5 29 49.60621	.01008	-0 45 2.9412	.0388	0.038674	0.003067	-2.54464	0.02140
1056	0	22	11.17	5 29 51.69993	.00842	-0 55 27.4665	.0304	0.004564	0.003294	-0.63173	0.01802
1057	0	52	9.95	5 29 51.92105	.00843	-0 44 43.0105	.0385	0.102275	0.002982	-3.60110	0.01389
1058	0	25	10.85	5 29 54.09684	.00995	-0 32 30.1159	.0421	-.044111	0.003544	1.01977	0.01673
1060	0	50	9.25	5 30 8.23984	.00795	-0 40 10.2103	.0295	-.012744	0.003358	0.80363	0.01420
1061	0	37	11.10	5 30 10.59044	.01085	-0 29 19.4708	.0306	0.019017	0.004822	1.17031	0.01330
1062	50060	99	7.50	5 30 16.75868	.00378	-0 44 50.8535	.0145	-.048927	0.001980	0.44150	0.00763
1063	0	35	10.10	5 30 18.92326	.01022	-0 9 21.9666	.0310	-.073605	0.005645	0.11607	0.01466
1064	0	53	9.67	5 30 29.07708	.00628	-0 45 58.2848	.0288	0.001520	0.002395	0.47958	0.01243
1065	0	31	11.27	5 30 36.33930	.01440	-0 54 31.8930	.0771	0.050462	0.004859	-3.50142	0.02468
1066	0	29	11.00	5 30 38.27483	.01150	-0 39 7.6461	.0349	0.032955	0.004932	-0.38675	0.01195
1067	0	76	9.15	5 30 42.84591	.00430	-0 57 18.0290	.0199	-.010343	0.002039	0.23167	0.00910
1068	0	10	10.35	5 30 45.68184	.05294	-0 52 8.1964	.2161	0.075690	0.011090	1.25063	0.03102
1069	0	56	9.85	5 30 49.09301	.00666	-0 22 4.9837	.0296	-.058424	0.002915	1.99911	0.01781
1070	50173	99	8.25	5 30 52.18552	.00405	-0 49 2.9400	.0198	0.024645	0.003101	1.26554	0.01240
1071	50187	64	8.45	5 30 54.30259	.00555	-0 28 29.8842	.0181	-.038142	0.003876	1.47944	0.01303
1072	0	30	11.00	5 31 1.17960	.00951	-0 35 0.2096	.0398	0.002434	0.005007	-1.43932	0.02414
1073	50215	87	8.40	5 31 4.54875	.00436	-0 17 43.9247	.0209	-.086974	0.003562	2.24821	0.01826
1074	50236	67	7.85	5 31 11.76780	.00458	-0 3 44.0934	.0148	-.006287	0.003852	0.95336	0.01068

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
1075	50260	78	8.20	5 31 18.10381	.00660	-0 6 35.6951	.0248	0.030132	0.003909	-0.12313	0.01443
1076	50271	59	9.30	5 31 20.95543	.01062	-0 8 20.3643	.0327	-.028207	0.004607	-0.33596	0.01344
1077	0	98	9.40	5 31 22.79320	.00802	-0 16 41.5661	.0233	-.002446	0.003780	-3.23684	0.01267
1078	0	36	10.85	5 31 26.69252	.00933	-0 5 48.0540	.0438	-.068134	0.002092	-0.75860	0.01669
1079	0	40	11.05	5 31 27.46959	.01031	-0 46 54.4790	.0296	-.028716	0.007508	-0.30491	0.02124
1080	0	36	10.15	5 31 28.63423	.01183	-0 2 51.5011	.0211	0.011162	0.004598	0.94782	0.01049
1081	50293	96	7.55	5 31 29.06599	.00546	-0 30 33.6604	.0197	-.022605	0.004544	-1.07523	0.01707
1082	50307	138	8.40	5 31 32.42473	.00498	-0 18 13.0357	.0146	0.002168	0.003085	-0.83626	0.01024
1083	0	39	10.40	5 31 35.19523	.01166	-0 26 1.9737	.0433	-.218337	0.003366	-0.74472	0.01819
1084	0	25	10.65	5 31 35.47799	.02130	-0 43 35.7058	.0606	-.186244	0.003717	-0.56182	0.01997
1085	0	16	10.50	5 31 35.84661	.01758	-0 40 45.9441	.0391	0.025845	0.005434	-3.41520	0.02866
1086	0	53	10.70	5 31 36.84955	.00960	-0 17 37.5942	.0352	-.053293	0.004240	-4.10070	0.02085
1087	0	56	10.80	5 31 40.88757	.00822	-0 23 1.9496	.0241	0.076160	0.003668	-0.89639	0.00745
1088	0	30	10.30	5 31 50.31120	.00792	-0 45 16.7023	.0268	0.013369	0.004112	0.12538	0.02042
1089	0	40	10.57	5 31 51.57208	.00812	-0 57 15.7172	.0229	-.012518	0.003796	-2.45052	0.01052
1090	0	39	11.05	5 31 51.68769	.00827	-0 57 53.9538	.0256	0.087897	0.004781	-0.58261	0.01267
1091	50370	140	8.20	5 31 52.02601	.00354	-0 47 44.9029	.0114	0.031577	0.003268	-0.18201	0.00896
1092	50377	140	8.70	5 31 52.97623	.00495	-0 24 49.7975	.0160	0.296999	0.004104	-7.77650	0.01384
1093	0	35	10.85	5 31 54.70880	.00978	-0 41 49.8022	.0309	-.029279	0.005579	-1.45484	0.02435
1094	50387	46	7.60	5 31 55.58789	.00880	-0 2 40.9379	.0316	-.008577	0.005223	0.95029	0.02088
1095	0	97	9.45	5 31 55.67801	.00511	-0 44 18.1164	.0150	0.019142	0.003090	0.18926	0.01043
1096	50396	144	8.20	5 32 0.42811	.00405	-0 25 8.2004	.0156	-.024034	0.003180	-1.36208	0.01131
1097	0	42	11.00	5 32 0.38670	.00948	-0 58 5.4301	.0190	0.003926	0.003873	-0.26436	0.01326
1098	0	110	8.50	5 32 4.23254	.00461	-0 54 56.7806	.0195	-.022139	0.001950	0.20517	0.00735
1099	0	43	10.60	5 32 13.27465	.00634	-1 0 25.2332	.0296	-.046715	0.002923	-0.84625	0.02091
1100	50436	112	7.97	5 32 15.60021	.00522	-0 59 9.9289	.0189	-.071916	0.002378	3.68985	0.00939
1101	0	29	10.25	5 32 19.43772	.01426	-0 12 30.6248	.0324	0.023971	0.004499	-1.33993	0.00814
1102	514731	10	9.20	5 32 22.29245	.04845	-0 9 14.3109	.3109	-.018836	0.014920	-0.13152	0.08868
1103	0	74	9.05	5 32 22.26939	.00706	-0 38 53.9046	.0258	0.046886	0.003492	0.08644	0.00972
1104	50468	39	7.00	5 32 22.90873	.02182	-0 9 16.9558	.0717	-.000088	0.007192	0.12645	0.02512
1105	50485	107	7.85	5 32 27.51330	.00556	-0 50 50.6429	.0277	-.022085	0.002632	-0.15565	0.01108
1106	50515	87	7.50	5 32 35.77744	.00695	-0 18 4.8867	.0297	0.009468	0.003555	-0.91440	0.01781
1107	0	89	8.45	5 32 37.75500	.00557	-0 52 6.8687	.0255	0.010809	0.002398	-0.42875	0.01059

Table 9: (Continued)

AC#	ACRS#	N	m	α			ϵ_α	δ			ϵ_δ	μ_α	$\epsilon_{\mu\alpha}$	μ_δ	$\epsilon_{\mu\delta}$
1108	0	39	11.10	5	32	38.05741	.00722	-0	56	38.0059	.0255	0.016662	0.003219	-0.90072	0.00900
1109	514743	34	10.87	5	32	38.83841	.01072	-0	56	24.3303	.0292	0.008997	0.005529	0.00135	0.01288
1110	50527	91	6.85	5	32	39.90434	.00795	-0	46	0.9472	.0321	0.002117	0.003832	-1.03838	0.01421
1111	0	31	10.20	5	32	43.77562	.00950	-0	37	15.6619	.0332	-.007251	0.003440	-0.02336	0.01566
1112	0	51	9.45	5	32	53.32185	.00848	-0	24	17.6593	.0439	-.041728	0.004834	-0.74900	0.01202
1113	0	25	10.70	5	32	58.24434	.01025	-0	30	8.8932	.0224	-.131161	0.005016	-1.16770	0.01282
1114	0	21	9.20	5	33	3.79032	.01218	-0	32	13.1857	.0317	-.068210	0.008442	0.22049	0.01483
1115	0	72	8.30	5	33	12.34825	.00420	-0	55	56.5517	.0235	-.008121	0.001907	-0.05583	0.00958
1116	0	24	10.35	5	33	13.84886	.02987	-0	6	44.7418	.0764	-.076600	0.006029	-0.19442	0.01068
1117	50682	81	7.70	5	33	15.44550	.00686	-1	1	4.1213	.0419	0.003533	0.002665	0.32750	0.01580
1118	0	36	10.40	5	33	16.01123	.01413	-0	23	30.4374	.0807	-.067534	0.004750	-0.55297	0.02667
1119	50683	81	8.30	5	33	16.31678	.00746	-1	1	12.4862	.0234	0.012354	0.003011	0.05658	0.00947
1120	0	25	10.25	5	33	22.21730	.00979	-0	44	42.3888	.0422	-.026552	0.003191	0.03094	0.01708
1121	0	68	8.70	5	33	22.68165	.00563	-0	48	29.8731	.0179	0.008687	0.003078	-0.05319	0.01112
1122	514784	43	9.20	5	33	24.26229	.01661	-0	20	47.0782	.0445	-.050707	0.008031	-0.97137	0.03224
1123	0	26	10.70	5	33	25.86318	.01188	-0	20	32.4567	.0440	-.124249	0.002517	-3.07005	0.01764
1124	50731	82	7.65	5	33	30.75800	.00477	-0	48	37.8502	.0191	0.032653	0.004210	0.07757	0.01304
1125	50737	67	8.30	5	33	32.49902	.01002	-0	21	31.3371	.0273	-.146703	0.004706	2.43874	0.02368
1126	0	51	9.20	5	33	41.97456	.00672	-0	26	34.6321	.0295	-.145859	0.004439	-1.08290	0.01270
1127	514796	19	10.30	5	33	42.14287	.05719	-0	25	36.5399	.0860	-.172653	0.028505	-1.40656	0.04289
1128	0	37	9.95	5	33	43.57281	.01615	-0	23	25.1343	.0478	0.105758	0.007450	-1.51474	0.01793
1129	50783	63	8.30	5	33	45.36392	.00941	-0	20	2.0296	.0397	-.095698	0.006569	1.61050	0.02262
1130	0	34	10.35	5	33	47.68540	.01321	-0	19	3.9593	.0577	-.062003	0.004274	-0.06538	0.01729
1131	0	3	10.65	5	33	52.72891	.11685	-0	3	7.8258	.6761	0.073248	0.019129	-3.72330	0.09425
1132	0	25	10.60	5	33	53.13393	.00867	-0	28	35.9255	.0291	-.101579	0.002978	-0.21450	0.01302
1133	0	60	9.95	5	33	54.57715	.00585	-0	50	13.8275	.0226	0.040608	0.004314	-0.30804	0.01046
1134	0	25	10.35	5	33	56.72597	.00861	-0	38	13.3215	.0291	-.007839	0.002984	0.29158	0.01418
1135	0	32	10.45	5	34	0.63143	.02046	-0	21	52.9045	.0568	-.084932	0.005994	0.06129	0.01379
1136	0	28	10.95	5	34	2.87530	.01459	-0	23	47.3823	.0327	-.097582	0.003613	-0.13852	0.01120
1137	0	45	10.05	5	34	5.39028	.00761	-0	52	24.6131	.0267	0.055537	0.003413	0.55979	0.01153
1138	0	27	10.10	5	34	10.99235	.01556	-0	19	54.1967	.0446	-.082604	0.004765	0.23014	0.00881
1139	50876	59	8.45	5	34	11.72933	.00771	-0	25	27.3093	.0246	-.162528	0.003885	0.35923	0.01187
1140	50913	67	7.10	5	34	24.44912	.00506	-0	43	53.1061	.0134	0.023122	0.003041	-0.34431	0.00756

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
1141	50962	43	7.83	5 34 38.54819	.01616	-0 13 31.4486	.0486	-.148559	0.006185	1.06840	0.01668
1142	50961	83	7.53	5 34 38.59799	.00612	-0 29 18.8052	.0161	-.165485	0.003895	-0.26776	0.01105
1143	0	60	8.70	5 34 39.01498	.00754	-0 24 42.4486	.0237	-.150892	0.004146	0.60691	0.01007
1144	50983	61	9.70	5 34 42.78975	.00763	-0 15 40.6319	.0214	-.131271	0.004985	1.78914	0.01071
1145	0	35	10.37	5 34 43.19398	.00926	-0 21 33.2946	.0351	-.070647	0.004966	0.14434	0.01454
1154	514848	73	7.40	5 34 50.30468	.00676	-0 43 35.2312	.0173	0.014803	0.004392	0.13969	0.00944
1155	51031	58	7.35	5 34 56.82603	.00773	-0 16 9.1287	.0259	-.163562	0.005932	1.04721	0.01959
1156	0	67	9.17	5 34 56.27486	.00694	-0 57 31.2470	.0224	-.032940	0.003251	0.76060	0.00950
1157	0	39	9.05	5 34 58.80909	.00867	-0 23 27.2680	.0386	-.135599	0.006205	0.56455	0.01872
1158	0	33	10.00	5 35 3.37682	.00816	-0 51 17.0485	.0276	0.013038	0.004686	0.65352	0.01454
1159	51052	94	7.07	5 35 3.99628	.00520	-0 54 57.1185	.0215	-.006279	0.002386	0.51555	0.00896
1160	0	8	11.00	5 35 6.97399	.05862	-0 18 28.9556	.2063	0.002005	0.012998	0.20966	0.07164
1161	51087	54	6.35	5 35 13.05479	.00789	-0 48 24.2971	.0231	-.010320	0.005310	-0.00269	0.01328
1162	51092	10	10.60	5 35 14.40054	.03587	-0 23 1.8877	.2421	0.042799	0.013848	-0.67711	0.05163
1163	0	20	9.35	5 35 13.88274	.01346	-0 35 59.7366	.0353	0.015248	0.004969	0.52183	0.01054
1170	0	23	10.73	5 36 2.05741	.01418	-1 0 44.0918	.0449	-.028119	0.004478	-0.03213	0.01428
1171	0	6	10.80	5 36 8.17673	.02619	-0 41 30.4539	.0947	-.044763	0.003816	-0.43897	0.02991
1173	0	6	10.95	5 36 41.22824	.01995	-0 26 45.6988	.0646	-.017337	0.004922	-0.42208	0.03189
1174	0	32	9.45	5 36 41.40757	.01693	-0 53 0.8720	.0579	-.037513	0.006289	0.39270	0.01875
1175	0	30	9.63	5 36 42.81469	.00898	-0 58 41.7140	.0357	-.025254	0.002705	-0.18587	0.01203
1176	0	13	9.55	5 36 47.31461	.01296	-0 47 15.1064	.0535	-.006851	0.004498	0.04856	0.02052
1177	0	8	8.65	5 37 2.34188	.02035	-0 25 23.1048	.0684	-.028953	0.005079	0.07020	0.04190
1179	0	6	10.30	5 37 12.20977	.02385	-0 28 57.1361	.1016	-.059963	0.013450	-0.64588	0.05968
1180	0	6	10.55	5 37 27.70251	.02323	-0 34 5.6855	.0998	-.174640	0.004781	-4.40859	0.03025
1182	51566	38	7.20	5 37 34.07098	.01341	-0 43 50.2698	.0304	-.085072	0.006932	0.66350	0.01890
1183	0	6	10.60	5 37 35.54861	.02357	-0 25 22.7131	.0855	-.092559	0.011654	-0.33274	0.03940
1185	51604	18	9.05	5 37 44.21003	.01150	-0 46 9.0925	.0400	-.014750	0.005123	0.41386	0.01725
1186	0	11	9.45	5 37 51.46261	.02018	-0 47 47.8166	.0340	-.049923	0.006000	0.23713	0.00740
1187	0	3	11.05	5 37 57.84388	.05597	-0 36 26.4325	.1383	-.067145	0.015749	-0.86249	0.02471
1188	0	6	10.45	5 37 58.80188	.02544	-0 24 21.4092	.0555	-.008959	0.003746	-0.10897	0.02643
1190	0	5	8.90	5 38 6.14633	.03634	-0 19 29.5256	.0749	-.026635	0.011922	-0.17851	0.02683
1232	0	43	10.80	5 22 29.03287	.00687	-0 54 59.2369	.0218	0.004925	0.003158	-1.01195	0.01363
1233	0	15	10.95	5 14 10.85257	.01130	-1 25 22.6652	.0535	-.159654	0.007904	-1.08641	0.02477

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon_{\mu\alpha}$	μ_δ	$\epsilon_{\mu\delta}$
1234	0	32	10.47	5 14 13.84698	.00669	-1 54 35.9629	.0249	-.036739	0.003692	-0.74733	0.01190
1235	0	30	10.95	5 14 24.33657	.00644	-1 52 35.8463	.0250	0.021876	0.003881	-1.58678	0.01380
1236	0	51	8.55	5 14 28.48140	.00517	-1 43 49.9245	.0175	-.075319	0.003013	0.34747	0.00918
1237	0	29	8.60	5 14 36.76319	.01147	-1 27 58.8133	.0246	-.030466	0.006773	0.94117	0.00813
1238	0	22	10.10	5 14 40.29100	.00922	-1 46 42.3064	.0264	-.020522	0.003720	-1.37170	0.01369
1239	47265	53	8.30	5 14 48.87820	.01092	-1 24 32.9485	.0246	-.014352	0.004863	-7.56978	0.01604
1240	0	9	11.15	5 14 50.98321	.04493	-1 7 30.7274	.1321	-.043017	0.008949	-2.48482	0.02115
1241	0	61	8.95	5 14 48.78373	.00498	-1 47 27.2041	.0169	-.022987	0.003023	-2.66767	0.00876
1242	0	17	11.05	5 14 52.84771	.02453	-1 19 59.9761	.1076	0.001563	0.009815	-1.90618	0.04616
1243	0	54	9.10	5 14 59.73241	.00507	-1 51 30.9760	.0176	-.104158	0.003700	-0.22343	0.01034
1244	0	13	11.25	5 15 6.21991	.03653	-1 1 38.6502	.0993	0.000225	0.008725	-0.13722	0.02455
1245	0	22	10.65	5 15 10.09898	.00806	-1 44 4.2097	.0254	-.072565	0.003922	0.52142	0.01397
1246	0	25	10.95	5 15 15.33124	.01690	-0 59 25.3691	.0526	-.013737	0.004520	1.95435	0.01788
1247	0	27	10.20	5 15 24.57274	.02073	-1 0 17.0941	.0655	-.015392	0.005426	-0.02918	0.01702
1248	0	21	11.15	5 15 22.35678	.00799	-1 41 13.7056	.0242	-.110847	0.004805	0.41784	0.01306
1249	0	19	10.45	5 15 25.22073	.02792	-1 4 44.6828	.0621	0.060290	0.006908	-1.76812	0.00864
1250	0	11	11.35	5 15 25.67636	.03958	-1 0 34.1805	.1749	-.048065	0.007228	-2.57854	0.03755
1251	0	8	11.45	5 15 27.81242	.06096	-1 0 57.9518	.0730	-.135398	0.014532	-2.44555	0.02121
1252	0	28	10.35	5 15 29.98291	.02020	-1 12 48.4973	.0604	-.028227	0.006419	0.22372	0.01646
1253	0	30	10.60	5 15 33.69587	.00964	-1 50 24.9833	.0337	-.089444	0.003223	-0.45801	0.01511
1254	0	48	9.85	5 15 42.81032	.00469	-1 51 16.3622	.0224	-.039453	0.002799	0.36335	0.01132
1255	0	22	10.65	5 15 50.23629	.00749	-1 42 58.3066	.0282	-.111930	0.003586	-0.28618	0.01302
1256	0	41	9.55	5 15 54.15263	.00726	-1 39 1.3913	.0315	0.180700	0.004063	-0.59509	0.01768
1257	0	54	8.90	5 15 56.37802	.01067	-1 7 37.7127	.0432	-.007129	0.003088	2.55349	0.01653
1258	47474	84	8.30	5 15 57.55512	.00986	-1 11 24.3542	.0354	-.000267	0.004335	2.77280	0.01487
1259	0	19	11.30	5 15 58.99591	.01855	-1 13 4.2144	.0958	-.036249	0.005974	-0.10040	0.03508
1260	0	78	8.70	5 15 59.94953	.00814	-1 19 2.7426	.0267	-.033551	0.002483	0.80385	0.01530
1261	0	25	10.85	5 16 8.64724	.02052	-1 17 23.3482	.0523	-.075492	0.008961	-0.26566	0.00889
1262	0	34	10.85	5 16 8.98604	.00768	-1 31 51.6710	.0347	-.114113	0.004997	-0.11469	0.01625
1263	0	39	10.35	5 16 18.57333	.01791	-1 11 14.3741	.0636	-.028533	0.005855	-0.65783	0.02436
1264	0	17	11.00	5 16 19.78462	.02106	-1 31 13.6567	.0666	0.152427	0.005671	-4.51323	0.03006
1265	0	33	10.70	5 16 24.23809	.01587	-1 21 13.4355	.0417	-.085958	0.006049	0.71360	0.01162
1266	0	37	10.95	5 16 29.44272	.02276	-1 2 37.1191	.0610	0.043844	0.007872	0.15188	0.02065

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon_{\mu\alpha}$	μ_δ	$\epsilon_{\mu\delta}$
1267	47588	76	7.75	5 16 35.03677	.01183	-1 8 59.3666	.0375	-.118144	0.004957	1.21981	0.01511
1268	0	94	9.10	5 16 33.82585	.00510	-1 52 50.1365	.0186	-.099151	0.003811	0.31388	0.01383
1269	0	17	11.05	5 16 42.60722	.01669	-1 1 51.2041	.0857	-.017852	0.003545	0.07909	0.01701
1270	0	33	10.50	5 16 48.66474	.01009	-1 27 44.0680	.0336	-.027656	0.008499	-0.07633	0.03042
1271	0	88	8.95	5 16 49.29763	.00828	-1 24 49.1921	.0196	-.033564	0.005027	0.61842	0.01358
1272	0	23	10.95	5 16 52.85570	.01245	-0 58 58.8723	.0451	-.026419	0.004286	0.13841	0.01346
1273	0	28	10.05	5 17 1.91424	.01480	-1 2 17.9213	.0618	-.008438	0.004430	-0.69529	0.01785
1274	0	30	10.15	5 17 2.95591	.02256	-1 9 52.8678	.0395	-.045888	0.007590	-3.37080	0.01818
1275	0	30	10.40	5 17 2.97581	.02385	-1 9 52.7966	.0735	-.058951	0.008357	-3.42377	0.02993
1276	47666	60	5.15	5 17 3.18147	.00812	-1 27 44.1169	.0345	-.017012	0.005122	0.71566	0.01887
1277	0	23	10.55	5 17 5.44336	.01882	-1 7 9.6341	.0627	-.081701	0.005409	0.44006	0.01164
1278	47673	70	7.55	5 17 6.32568	.01161	-1 9 31.4153	.0364	-.045364	0.004641	-1.02907	0.01509
1279	47678	104	8.45	5 17 7.95185	.00588	-1 24 23.5130	.0158	-.036105	0.004210	0.49964	0.01160
1280	47680	87	8.00	5 17 8.49841	.00607	-1 33 59.7923	.0193	-.089021	0.003580	-0.75348	0.01035
1281	0	34	10.90	5 17 9.41569	.01195	-1 20 48.6568	.0383	-.033642	0.005082	-0.72148	0.01957
1282	0	16	11.00	5 17 10.43084	.02407	-1 8 40.7643	.1067	-.044225	0.007702	-0.28974	0.02253
1283	0	33	10.80	5 17 14.10248	.01081	-1 20 17.0045	.0380	0.039506	0.006020	1.02727	0.02470
1284	514057	29	10.10	5 17 16.80718	.01791	-1 7 3.1816	.1220	0.046480	0.006290	0.45026	0.04449
1285	0	34	10.20	5 17 23.77975	.01043	-1 22 20.3075	.0313	0.001157	0.004455	-0.55930	0.01951
1286	0	46	9.75	5 17 26.55086	.01014	-1 17 32.5381	.0362	0.008118	0.004907	-1.27477	0.01499
1287	0	55	9.55	5 17 25.48766	.00712	-1 37 59.4673	.0202	-.046320	0.003163	-0.09164	0.01303
1288	0	19	10.65	5 17 27.02039	.02344	-1 16 41.9671	.0833	-.043938	0.007625	-0.98817	0.02316
1289	514064	48	8.40	5 17 28.46283	.02081	-1 3 9.6634	.0756	0.026496	0.007963	-0.51318	0.02997
1290	0	26	10.60	5 17 33.06844	.01076	-1 31 36.4723	.0287	-.037622	0.006371	-0.54207	0.01389
1291	0	41	9.10	5 17 40.73914	.00784	-1 31 49.4374	.0295	-.081772	0.001561	0.16391	0.01261
1292	0	10	10.85	5 17 46.79907	.03746	-1 7 40.4556	.1088	0.167918	0.007561	0.02259	0.03334
1293	0	32	9.33	5 17 44.87342	.00935	-1 59 56.5187	.0362	-.145733	0.004699	-1.03714	0.01690
1294	0	25	10.97	5 17 48.11405	.01288	-1 54 49.4811	.0578	-.073991	0.005962	-1.94509	0.03656
1295	0	25	10.25	5 17 51.13738	.01891	-1 28 39.5520	.0429	0.081166	0.005186	-3.02999	0.01855
1296	0	18	11.10	5 17 55.53589	.01623	-1 19 10.2429	.0577	0.011643	0.006621	-0.05972	0.02239
1297	0	29	10.30	5 17 55.10323	.00858	-1 33 16.7747	.0276	0.028111	0.004946	-1.72474	0.01815
1298	0	19	10.90	5 17 56.77702	.01219	-1 26 44.5486	.0458	-.004758	0.003343	-1.79543	0.02834
1299	0	20	10.75	5 18 2.90565	.01243	-1 26 50.8763	.0513	0.031728	0.005810	0.30898	0.02057

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
1300	0	17	11.45	5 18 1.90533	.01453	-1 39 56.5241	.0361	-.188055	0.004764	-0.04328	0.01357
1301	0	19	10.30	5 18 5.76916	.03367	-1 11 55.6365	.0863	-.005768	0.009241	0.15408	0.01349
1302	0	9	11.50	5 18 8.02930	.03911	-1 15 34.0727	.1531	0.011085	0.006965	-1.85833	0.02840
1303	0	17	11.40	5 18 12.25671	.02249	-1 22 41.7402	.0556	0.009623	0.008678	-0.39874	0.01419
1304	0	21	10.80	5 18 14.71804	.01280	-1 22 22.8142	.0568	-.054810	0.006460	-0.37404	0.02871
1305	0	38	10.57	5 18 16.15477	.00890	-1 28 59.7945	.0250	0.041841	0.002695	-2.82808	0.01224
1306	0	8	11.45	5 18 22.23246	.04804	-1 7 47.0890	.1736	-.036725	0.010553	-0.66114	0.05043
1307	0	8	11.45	5 18 22.26909	.03801	-1 7 46.4202	.1815	-.046344	0.009785	-1.26847	0.04654
1308	0	39	10.50	5 18 28.11493	.01677	-1 31 17.0687	.0457	0.042663	0.004972	-3.12657	0.01627
1309	0	22	11.35	5 18 33.72032	.01568	-1 28 22.6870	.0497	-.039429	0.004296	-0.25436	0.02243
1310	0	8	11.20	5 18 39.44626	.03836	-1 5 54.2706	.0910	-.005052	0.010256	-1.39456	0.02005
1311	0	31	10.90	5 18 38.76551	.01058	-1 27 59.2531	.0306	-.026358	0.004035	-1.56329	0.01646
1312	0	57	9.07	5 18 40.45697	.00653	-1 47 38.0960	.0226	-.112973	0.002661	-0.81070	0.00982
1313	0	26	11.10	5 18 45.20516	.00997	-1 57 59.0285	.0452	-.043424	0.005442	-2.00043	0.02587
1314	48006	71	6.30	5 18 56.47334	.00562	-1 35 39.6639	.0286	0.035313	0.003543	-0.33299	0.01268
1315	0	30	11.15	5 19 4.12702	.01218	-1 30 2.1797	.0337	-.069499	0.005425	-1.85464	0.01531
1316	0	31	10.95	5 19 5.62862	.00863	-1 50 5.0522	.0357	-.074845	0.005867	0.04985	0.02091
1317	0	11	11.15	5 19 8.53787	.02215	-1 10 44.7904	.0753	0.068410	0.005967	0.96246	0.02021
1318	0	30	10.60	5 19 9.75457	.00797	-1 46 13.7585	.0276	0.004187	0.003915	-1.30091	0.01086
1319	0	23	11.25	5 19 19.02751	.01125	-1 30 55.9049	.0407	-.062463	0.002706	0.01539	0.01723
1320	0	34	10.65	5 19 24.95980	.00854	-1 40 3.8880	.0307	0.014404	0.003367	-1.08372	0.01254
1321	0	38	10.00	5 19 26.06206	.01358	-1 50 13.0957	.0300	-.085865	0.005516	0.40131	0.00990
1322	0	38	11.25	5 19 27.22559	.00736	-1 45 3.0220	.0284	0.082183	0.001620	-3.47308	0.01523
1323	0	41	11.05	5 19 32.16807	.00815	-1 48 43.4234	.0238	-.041344	0.003741	-1.42752	0.01217
1324	0	27	10.90	5 19 37.04135	.00979	-1 52 5.0299	.0320	0.167254	0.005151	-3.87928	0.01107
1325	0	36	10.45	5 19 38.85225	.00833	-1 28 27.4843	.0233	-.065924	0.004630	-0.86871	0.01424
1326	0	36	10.90	5 19 39.01342	.00879	-1 33 10.3482	.0213	-.083787	0.003673	0.38089	0.00846
1327	0	30	10.30	5 19 40.94203	.01273	-1 13 18.4896	.0353	-.079836	0.005065	-0.23481	0.00770
1328	0	22	11.25	5 19 41.96511	.01286	-1 26 52.9206	.0323	-.042360	0.002339	-0.27567	0.01030
1329	0	36	10.30	5 19 40.68054	.01249	-1 55 31.7965	.0396	-.040879	0.004975	-0.25450	0.01404
1330	0	28	10.80	5 19 43.54747	.00811	-1 25 24.6308	.0227	-.177413	0.002185	-2.03603	0.00873
1331	0	25	11.20	5 19 44.68502	.01444	-1 39 48.5298	.0331	-.021120	0.005237	-1.53167	0.01368
1332	48184	53	8.10	5 20 6.30767	.00530	-1 5 21.9627	.0204	-.055241	0.004476	0.65219	0.01232

Table 9: (Continued)

AC#	ACRS#	N	m	α			ϵ_α	δ			ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
1333	0	21	10.45	5	20	7.60992	.00773	-1	3	48.4895	.0288	-.095776	0.002350	-2.78557	0.01557
1334	0	20	11.05	5	20	11.65589	.00741	-1	5	40.0368	.0249	-.149990	0.004367	-0.94845	0.01295
1335	0	32	9.65	5	20	12.80057	.01078	-1	34	49.5938	.0400	0.020249	0.004837	-2.03244	0.01785
1337	0	19	11.25	5	20	29.81971	.00851	-1	6	10.7452	.0245	-.022556	0.004836	-0.33018	0.01201
1338	0	33	9.40	5	20	30.96806	.00596	-1	17	59.5677	.0266	-.120392	0.003521	0.72181	0.01164
1339	0	20	11.10	5	20	47.03477	.00928	-1	35	59.5435	.0309	-.028328	0.004114	-1.03743	0.01355
1340	48315	54	7.35	5	20	50.86654	.00448	-1	29	12.4293	.0145	-.098107	0.002584	-0.37212	0.00851
1341	0	3	11.25	5	20	54.20982	.20962	-1	22	30.9857	.3743	-.020957	0.029627	-1.12222	0.05843
1342	48325	61	7.00	5	20	55.58519	.00667	-1	2	53.3342	.0350	-.001429	0.004739	0.65576	0.01611
1343	0	51	9.40	5	20	58.51259	.00697	-1	7	8.0909	.0255	-.037618	0.004326	-0.28075	0.01531
1344	0	21	10.90	5	20	58.80664	.00638	-1	9	56.8962	.0322	-.040207	0.004331	-0.55948	0.01235
1345	0	28	11.35	5	21	11.23210	.01028	-1	13	14.1712	.0387	-.058290	0.004192	0.84864	0.01916
1346	0	22	11.30	5	21	10.51435	.01276	-1	30	36.3225	.0458	-.080877	0.004416	-0.66546	0.02029
1347	0	25	11.10	5	21	14.36319	.01071	-1	0	38.8481	.0354	-.030294	0.007166	0.29944	0.02207
1348	0	33	9.85	5	21	14.17128	.00992	-1	29	9.4291	.0305	-.058132	0.004902	0.62415	0.01466
1349	0	25	11.35	5	21	18.81051	.00950	-1	1	45.5360	.0351	-.027045	0.006666	-0.66541	0.02881
1350	0	55	9.10	5	21	26.25824	.00711	-1	26	48.5493	.0260	-.110744	0.002832	0.74903	0.00848
1351	0	52	9.05	5	21	26.94615	.00817	-1	27	36.0437	.0326	-.097082	0.003410	-2.51457	0.01566
1352	0	32	11.30	5	21	38.33031	.01505	-1	30	54.1754	.0645	-.065582	0.004993	0.17649	0.02263
1353	0	27	11.05	5	21	46.57861	.00908	-0	59	1.2469	.0340	-.109389	0.004966	0.66270	0.02309
1354	0	48	9.05	5	21	45.34463	.00609	-1	40	39.9116	.0350	-.052175	0.002922	-1.27803	0.01480
1355	0	41	11.40	5	22	6.15431	.00851	-1	17	1.6255	.0613	-.011070	0.004639	-0.74388	0.01943
1356	0	32	11.50	5	22	9.52549	.01039	-1	22	15.6013	.0376	-.084982	0.002513	-0.84495	0.01750
1357	0	40	10.97	5	22	16.01078	.00824	-1	16	57.2830	.0323	0.018347	0.003865	-2.02746	0.01355
1358	0	25	10.60	5	22	19.88262	.00958	-1	39	34.1411	.0344	-.034779	0.005099	-2.61216	0.01486
1359	0	55	10.47	5	22	25.97083	.00601	-1	11	25.7322	.0249	0.003231	0.002140	-0.22684	0.01476
1360	0	37	11.27	5	22	32.78760	.00763	-0	58	0.2018	.0258	-.065547	0.003354	-1.04666	0.01851
1361	0	41	11.17	5	22	35.48701	.00833	-1	5	2.3993	.0262	-.067790	0.004781	-1.01023	0.01839
1362	0	50	10.83	5	22	32.72472	.00897	-1	48	43.1494	.0360	0.004524	0.004407	-0.22839	0.03046
1363	0	36	11.50	5	22	41.44107	.01159	-1	8	5.8568	.0526	-.048463	0.009049	-0.50610	0.01372
1364	0	116	8.93	5	22	43.29678	.00439	-1	16	8.6557	.0154	-.063552	0.002586	0.59313	0.00742
1365	0	31	11.10	5	22	56.40469	.00781	-0	58	9.9806	.0347	-.051471	0.002371	-0.13957	0.02930
1366	0	35	10.90	5	22	57.77851	.00968	-1	8	40.0996	.0409	-.038967	0.004212	-1.03797	0.03412

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
1367	48698	68	4.95	5 23 4.48681	.00590	-1 32 4.3719	.0304	-.128680	0.004498	-0.34310	0.02044
1368	0	26	10.60	5 23 9.05528	.01276	-1 56 29.5967	.0556	0.005674	0.010456	-4.33315	0.05050
1369	0	37	10.45	5 23 10.45373	.00770	-1 16 9.5393	.0322	-.079266	0.003859	-0.74363	0.01250
1370	0	30	10.00	5 23 16.74588	.01093	-1 5 27.9073	.0414	-.060733	0.004046	-0.37597	0.02289
1371	0	53	8.70	5 23 16.19565	.01041	-1 53 47.6394	.0426	-.098692	0.008451	-0.48036	0.03456
1372	0	23	11.35	5 23 31.48814	.01083	-1 32 31.7571	.0392	-.097525	0.007553	-1.77899	0.01603
1373	0	38	10.93	5 23 33.44775	.01035	-2 0 30.9437	.0373	-.029204	0.006931	-0.67892	0.02234
1374	0	34	10.40	5 23 41.19986	.00877	-1 45 51.6889	.0350	-.058594	0.003406	-0.98432	0.01464
1375	0	34	11.00	5 24 7.97789	.00818	-1 35 24.6639	.0313	-.061917	0.004313	-2.49380	0.01375
1376	48896	60	8.80	5 24 8.53806	.00992	-1 54 22.6712	.0325	-.236599	0.008478	-2.95567	0.02749
1377	0	21	11.10	5 24 13.23498	.00942	-0 58 22.9605	.0326	-.033414	0.004423	-1.37300	0.01694
1378	0	31	10.53	5 24 14.13565	.01022	-1 57 4.7966	.0433	-.025719	0.005273	-2.45029	0.02217
1379	0	39	10.45	5 24 16.71369	.00953	-1 19 42.5754	.0305	-.039639	0.005591	-1.98802	0.01952
1380	514356	87	8.80	5 24 16.91036	.00720	-1 33 5.6986	.0242	-.061335	0.003302	-0.85121	0.01117
1381	0	60	9.05	5 24 18.78362	.00603	-1 7 17.4722	.0268	-.047734	0.004327	-0.33455	0.01506
1382	0	32	11.25	5 24 18.86696	.01291	-1 21 50.4639	.0366	0.079274	0.002138	-2.14466	0.01631
1383	0	26	10.70	5 24 19.42319	.01063	-1 9 13.6322	.0412	-.051228	0.003717	-0.28952	0.02754
1384	0	41	10.20	5 24 25.18569	.00871	-1 14 55.9664	.0373	-.076357	0.004113	-0.71347	0.01882
1385	0	12	10.75	5 24 26.48400	.03603	-1 8 27.5924	.2012	-.119767	0.007076	-4.46417	0.03059
1386	0	40	10.55	5 24 28.60947	.00850	-1 20 26.7385	.0352	-.045478	0.001730	-0.17912	0.01242
1387	0	27	11.15	5 24 29.96716	.01077	-1 33 22.5336	.0311	-.035087	0.006484	-0.65645	0.01268
1388	48987	69	6.90	5 24 34.87527	.00609	-1 7 29.8551	.0302	-.159410	0.003757	4.94646	0.01827
1389	48992	64	9.00	5 24 36.69734	.00765	-1 52 21.3288	.0270	0.048585	0.004958	0.38067	0.01527
1390	48997	73	5.90	5 24 37.20862	.00510	-1 24 31.2271	.0283	-.069030	0.003337	-0.42902	0.01661
1391	0	34	10.50	5 24 43.42459	.00675	-1 38 34.3112	.0291	0.038142	0.003359	-0.15465	0.01158
1392	49029	26	10.25	5 24 47.23662	.00844	-1 9 32.6905	.0379	-.064514	0.004648	0.23159	0.02012
1393	0	30	10.75	5 24 47.99433	.00983	-1 38 41.8638	.0362	-.006424	0.004133	-1.31822	0.02169
1394	0	28	10.05	5 24 52.54596	.00664	-1 30 9.9537	.0349	-.033428	0.002787	-0.41148	0.00687
1395	0	22	11.15	5 24 56.35270	.01105	-1 36 30.7650	.0406	0.087964	0.002715	0.32341	0.02300
1396	0	51	9.50	5 25 3.68893	.00777	-1 47 34.1546	.0285	-.008324	0.003876	0.16971	0.01340
1397	0	20	11.05	5 25 5.99182	.00974	-1 39 23.5795	.0392	0.001107	0.005488	-0.36061	0.01976
1398	0	36	9.00	5 25 6.52359	.00731	-1 39 7.5996	.0325	-.005776	0.003479	-0.31438	0.01417
1399	49089	58	8.97	5 25 7.15963	.00670	-2 0 28.3118	.0233	0.000409	0.002713	0.22258	0.01107

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
1401	0	30	9.70	5 25 12.21992	.00783	-1 14 41.2287	.0315	-.057781	0.004320	-0.51396	0.01068
1402	49116	92	6.45	5 25 13.07486	.00566	-1 51 13.1983	.0263	-.036802	0.004107	-1.41908	0.01468
1403	0	40	9.85	5 25 16.17518	.00580	-1 15 4.8751	.0266	-.032872	0.003685	1.19410	0.01432
1404	0	39	8.45	5 25 19.40044	.00748	-1 16 30.1124	.0325	0.048997	0.004649	-0.53700	0.00823
1405	0	35	11.30	5 25 24.86023	.01295	-1 47 18.8627	.0354	0.043878	0.006724	0.31624	0.01430
1406	49183	82	6.85	5 25 34.68671	.00618	-1 42 32.9061	.0344	-.212771	0.003225	-7.37359	0.01563
1407	0	47	9.70	5 25 37.42005	.00679	-1 7 59.7841	.0251	-.030688	0.003197	-0.66696	0.01112
1408	49205	153	7.40	5 25 38.77866	.00348	-1 48 42.6328	.0142	-.030537	0.002930	-0.66457	0.00906
1409	49211	102	7.77	5 25 41.15157	.00506	-1 58 52.6060	.0153	-.016712	0.003110	-0.17735	0.01029
1410	49215	77	8.63	5 25 41.99342	.00618	-1 59 13.2124	.0205	-.006470	0.003691	0.22316	0.01166
1411	0	48	9.93	5 25 43.40636	.00751	-2 1 0.9645	.0243	-.016751	0.003015	-0.00726	0.01201
1412	0	33	11.35	5 25 49.28339	.00941	-1 31 38.9724	.0341	0.073594	0.004637	-0.72297	0.01451
1413	0	24	10.85	5 25 49.91300	.00932	-1 11 42.6603	.0273	0.065507	0.007090	-0.41011	0.01030
1414	0	31	11.35	5 25 51.94216	.00880	-1 34 35.4488	.0235	-.013148	0.002702	1.71646	0.01282
1415	0	22	11.00	5 25 56.04865	.00646	-1 4 36.8503	.0239	-.050891	0.003598	-1.09156	0.01064
1416	0	72	9.40	5 25 55.66254	.00507	-1 41 10.3124	.0209	-.001544	0.001524	0.06423	0.00948
1417	0	37	10.93	5 25 55.92608	.00642	-1 56 32.4036	.0247	0.076891	0.003575	-0.16526	0.01059
1418	0	51	11.00	5 25 58.93471	.00749	-1 50 13.0516	.0289	-.045201	0.006255	0.00201	0.01866
1419	0	25	10.60	5 26 1.80390	.01125	-1 15 27.3375	.0299	0.040878	0.005348	-0.87786	0.00925
1420	0	57	10.55	5 26 2.34861	.00763	-1 52 6.3900	.0277	-.015289	0.006478	-0.32052	0.01940
1421	0	43	10.85	5 26 7.12019	.00737	-1 25 35.5793	.0458	0.034812	0.003712	0.19082	0.01732
1422	0	33	11.20	5 26 20.15046	.00952	-1 51 48.4677	.0422	0.056398	0.004692	1.02301	0.02314
1423	0	26	11.20	5 26 20.60024	.01112	-1 32 58.9416	.0423	0.120779	0.005195	-0.90709	0.01776
1424	49345	93	8.13	5 26 23.47130	.00430	-1 47 59.8904	.0187	-.016989	0.002846	-0.26012	0.01255
1425	0	25	11.27	5 26 24.60098	.01084	-1 51 7.2003	.0331	0.076506	0.008377	0.28528	0.01956
1426	514463	84	8.50	5 26 33.63522	.00580	-1 41 21.4836	.0211	-.240717	0.003143	0.47368	0.01088
1427	0	30	10.40	5 26 41.33052	.00844	-1 36 52.8856	.0305	0.038913	0.005444	-0.07252	0.01319
1428	0	57	8.70	5 26 41.91728	.01138	-1 50 6.0276	.0267	0.130992	0.005891	0.66603	0.01285
1429	0	33	10.57	5 26 46.13587	.00813	-1 45 58.6614	.0300	0.034647	0.004614	-0.03330	0.01830
1430	0	24	11.00	5 26 46.56667	.01116	-1 20 56.5334	.0387	0.021399	0.004754	-0.22516	0.01580
1431	0	11	10.97	5 26 47.02075	.01702	-1 6 2.9789	.0718	-.033154	0.004604	-1.40739	0.02271
1432	0	34	10.25	5 26 50.75168	.00884	-1 34 12.9785	.0248	0.058381	0.005032	0.26236	0.01124
1433	0	30	11.15	5 26 50.94474	.01108	-1 28 18.6689	.0284	0.069782	0.004715	-0.66432	0.00967

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
1434	49451	30	8.70	5 27 1.03983	.01575	-1 13 37.0883	.0604	0.082197	0.004071	-2.51231	0.01206
1435	0	30	9.60	5 27 2.03901	.01358	-1 18 13.2608	.0703	0.007558	0.004221	-0.51987	0.02494
1436	49484	26	6.45	5 27 11.55086	.03214	-1 7 48.7077	.1260	0.026792	0.011355	-2.21617	0.04457
1437	0	22	10.55	5 27 12.35222	.02049	-1 2 52.1697	.0983	0.146241	0.006554	-0.38628	0.03557
1438	49488	19	9.25	5 27 12.37712	.02676	-1 7 47.9963	.0579	-.004211	0.009551	-2.46344	0.01988
1439	0	35	10.65	5 27 15.23849	.00913	-1 33 10.5183	.0353	0.050197	0.003941	0.42789	0.01592
1440	49499	84	7.50	5 27 17.08773	.00796	-1 41 45.4113	.0238	0.073366	0.003960	0.33329	0.01355
1441	0	32	8.95	5 27 17.74929	.01866	-1 4 26.0939	.0861	-.029402	0.006720	0.18291	0.03124
1442	0	31	8.95	5 27 25.24777	.01071	-1 4 7.1619	.0196	-.016582	0.003474	0.01722	0.00369
1443	0	40	10.20	5 27 25.47170	.00758	-1 30 27.5727	.0238	-.012941	0.003307	-0.02369	0.01072
1444	0	30	10.57	5 27 25.01679	.01179	-2 1 1.7684	.0398	0.105031	0.007051	1.14373	0.02009
1445	49534	61	9.05	5 27 27.87376	.00960	-1 37 41.8881	.0265	0.061262	0.003996	0.63520	0.01334
1446	49548	84	6.55	5 27 32.70103	.00599	-1 47 14.2486	.0315	0.190340	0.004129	0.65924	0.01479
1447	0	80	8.95	5 27 35.10330	.00603	-1 30 9.3691	.0211	0.016938	0.002608	1.35422	0.01050
1448	49563	50	8.10	5 27 37.67959	.01697	-1 3 56.5845	.0629	-.021350	0.006048	-0.24890	0.02226
1449	0	51	9.95	5 27 39.41129	.00659	-1 43 22.6822	.0285	0.011853	0.003206	-0.31699	0.01181
1450	0	32	10.85	5 27 41.24963	.01466	-1 10 14.0705	.0674	0.036504	0.004364	-2.36924	0.02361
1451	0	62	9.00	5 27 41.77471	.00593	-1 33 25.6251	.0191	0.065266	0.002690	0.43659	0.00914
1452	0	35	10.35	5 27 50.01215	.00913	-1 25 5.3650	.0278	0.069823	0.003991	0.68808	0.00980
1453	0	34	10.35	5 28 6.42649	.00852	-1 41 16.3598	.0213	0.020111	0.004009	0.07619	0.01019
1454	0	21	10.70	5 28 10.16715	.01477	-1 3 35.7314	.0514	-.001441	0.004624	-0.32338	0.01743
1455	0	34	11.00	5 28 26.42018	.01134	-1 59 5.0010	.0308	0.029569	0.004803	0.18960	0.01532
1456	0	41	10.75	5 28 30.76443	.00848	-1 18 4.8500	.0359	0.039174	0.004238	-0.55359	0.01543
1457	0	74	9.25	5 28 31.60435	.00977	-1 13 37.5391	.0420	0.105289	0.004047	-0.91763	0.01662
1458	0	50	10.85	5 28 34.35129	.00732	-1 27 17.5313	.0239	0.095441	0.004082	-0.70472	0.01114
1459	0	50	10.75	5 28 35.85341	.00636	-1 30 32.8715	.0175	0.022759	0.003001	-0.30914	0.00945
1460	0	46	10.55	5 28 37.44594	.00547	-1 40 24.4422	.0198	0.062120	0.002583	-0.48473	0.00802
1461	0	38	10.80	5 28 47.31275	.00633	-1 10 54.8337	.0249	0.032827	0.002749	-0.34470	0.01062
1462	49789	109	7.80	5 28 49.28263	.00486	-2 1 2.9176	.0133	-.002064	0.003430	0.17462	0.00949
1463	0	51	10.65	5 28 50.94435	.00695	-1 27 12.3050	.0201	0.034016	0.003194	-0.10309	0.01020
1464	0	53	10.75	5 28 51.07886	.00705	-1 19 54.0222	.0239	0.019724	0.005093	0.28750	0.00751
1465	0	52	10.10	5 28 55.23191	.00553	-1 30 45.8759	.0188	0.040820	0.003024	0.17614	0.01033
1466	0	92	9.40	5 28 57.50898	.00462	-1 46 11.7830	.0186	0.023450	0.002289	1.08477	0.00852

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
1467	49821	101	8.25	5 28 59.75005	.00465	-1 51 43.0676	.0151	0.031255	0.003416	0.48170	0.00919
1468	0	51	10.75	5 29 1.82913	.00865	-1 26 48.2099	.0221	-.006969	0.001549	0.18921	0.01131
1469	0	26	11.25	5 29 2.70537	.00758	-1 1 52.3561	.0310	0.035053	0.003885	-0.34311	0.01517
1470	0	69	9.90	5 29 3.30131	.00527	-1 42 14.2448	.0188	0.034188	0.002900	0.50202	0.00870
1471	49852	119	8.15	5 29 8.07171	.00423	-1 9 42.4424	.0180	0.003557	0.002138	0.52770	0.00923
1472	49866	90	8.20	5 29 11.34560	.00471	-1 54 45.1249	.0161	0.004324	0.003761	2.52692	0.01265
1473	0	41	10.65	5 29 12.73583	.00782	-1 33 7.2035	.0242	0.017071	0.003385	0.28460	0.01217
1474	0	32	11.15	5 29 13.66011	.01339	-1 41 57.3758	.0467	0.012560	0.005015	0.03658	0.02133
1475	0	78	8.55	5 29 14.61847	.00663	-1 37 17.3446	.0226	0.063800	0.002612	0.41867	0.00999
1476	0	42	10.55	5 29 16.05469	.00563	-1 27 5.2369	.0244	0.072465	0.002568	0.06985	0.01087
1477	0	39	10.85	5 29 16.53304	.00693	-1 41 49.0622	.0213	-.003680	0.002956	-0.75079	0.00702
1478	0	27	10.53	5 29 17.02014	.01015	-1 58 31.9174	.0359	0.039722	0.003930	-0.13085	0.01458
1479	0	50	10.80	5 29 17.81839	.00639	-1 21 2.7381	.0196	0.039985	0.003337	-1.22595	0.01134
1480	0	40	10.95	5 29 18.89081	.00862	-1 11 5.0069	.0336	-.006272	0.004781	0.44989	0.01815
1481	0	37	10.80	5 29 20.64518	.00721	-1 14 34.5236	.0219	0.024554	0.004579	-0.18366	0.01092
1482	0	35	10.90	5 29 21.69073	.00799	-1 36 23.6252	.0259	0.032727	0.003762	-0.05444	0.00969
1483	0	78	8.55	5 29 32.05108	.00507	-1 30 52.1749	.0193	-.019054	0.003032	-0.12373	0.01122
1484	49934	64	8.85	5 29 33.41892	.00680	-1 39 21.0338	.0279	0.030518	0.003086	0.11098	0.01167
1485	0	32	10.95	5 29 34.48345	.00816	-1 16 16.9743	.0301	0.012974	0.004344	-0.07145	0.01343
1486	0	45	9.70	5 29 36.65591	.00880	-1 3 36.4576	.0347	-.133584	0.004787	-3.12018	0.01490
1487	0	27	10.70	5 29 36.76087	.00774	-1 8 1.6457	.0285	-.004528	0.003561	0.02454	0.00828
1488	0	41	10.85	5 29 38.15259	.00757	-1 22 13.3389	.0233	-.025295	0.004969	0.27911	0.01270
1489	0	36	11.00	5 29 38.59651	.00668	-1 15 41.8431	.0208	0.001946	0.002556	0.10892	0.01048
1490	49956	83	7.70	5 29 41.22342	.00449	-1 38 8.5626	.0152	0.010178	0.003343	0.20329	0.01019
1491	0	38	9.57	5 29 42.37100	.00718	-1 59 30.4144	.0329	-.008275	0.003319	0.41546	0.01315
1492	0	32	10.25	5 29 43.07377	.00689	-1 48 55.9903	.0271	0.013145	0.003147	0.19378	0.01185
1493	0	37	10.35	5 29 47.56089	.00771	-1 26 19.3928	.0270	-.001885	0.003622	-0.55527	0.01135
1494	0	45	9.90	5 29 49.25789	.00691	-1 3 25.2875	.0228	-.003170	0.002768	-0.01845	0.01190
1495	0	54	9.03	5 29 52.10768	.00535	-1 55 8.0515	.0207	0.028831	0.002567	-0.20954	0.00981
1496	0	9	11.00	5 29 52.69961	.04525	-1 20 47.4630	.3086	0.084795	0.009071	-0.56116	0.04938
1497	0	28	11.10	5 29 57.23811	.00718	-1 2 44.3363	.0358	-.033351	0.003953	-0.30794	0.01575
1498	0	38	10.70	5 30 0.29348	.00679	-1 30 37.6998	.0252	-.009213	0.001678	0.04778	0.01077
1499	0	44	10.60	5 30 1.97860	.00518	-1 24 15.8628	.0234	-.007278	0.002866	0.30498	0.01008

Table 9: (Continued)

AC#	ACRS#	N	m	α			ϵ_α	δ			ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
1500	0	27	10.95	5	30	2.68250	.00799	-1	1	14.6586	.0300	0.003317	0.004897	-0.29982	0.01383
1501	0	37	10.10	5	30	2.45634	.00613	-1	12	39.4518	.0372	-.128060	0.001773	-1.88763	0.01030
1502	0	55	9.80	5	30	3.19887	.00770	-1	55	30.1780	.0279	0.043786	0.003303	-1.86212	0.01140
1503	0	22	11.15	5	30	6.49063	.00738	-1	5	57.9011	.0314	0.013325	0.003560	0.04942	0.01923
1504	0	36	10.55	5	30	6.63665	.00574	-1	51	31.7738	.0253	0.002232	0.002349	-0.31450	0.01192
1505	0	34	10.75	5	30	7.19865	.00919	-1	35	29.8537	.0290	-.034372	0.004219	-0.24419	0.01245
1506	0	45	10.20	5	30	9.05474	.00576	-1	19	57.7450	.0211	0.000644	0.004373	-0.21013	0.00745
1507	0	54	8.65	5	30	9.20023	.00545	-1	50	38.6998	.0236	0.041541	0.003248	-0.07061	0.01062
1508	50035	31	4.30	5	30	9.47686	.01765	-1	37	35.5086	.0946	-.024359	0.007337	1.28593	0.03649
1509	50052	77	8.45	5	30	13.66171	.00395	-1	4	2.8354	.0159	-.002457	0.002295	0.10666	0.01136
1510	0	62	8.53	5	30	18.56281	.00479	-1	38	6.7740	.0183	0.003138	0.002567	0.15069	0.00851
1511	0	82	9.00	5	30	27.98750	.00492	-1	22	14.1852	.0161	0.087014	0.003242	-0.24678	0.00916
1512	0	33	10.93	5	30	28.35253	.01881	-1	33	50.7585	.0385	0.020261	0.006048	0.16711	0.01589
1513	50104	94	7.50	5	30	31.38490	.00466	-1	16	30.8276	.0178	-.087328	0.003100	-0.12705	0.01016
1514	0	45	10.40	5	30	33.85049	.00829	-1	18	35.5431	.0272	-.007649	0.004688	-0.23618	0.01294
1515	50111	63	7.33	5	30	33.84257	.00768	-1	45	18.4163	.0218	-.066440	0.005611	-0.47165	0.01331
1516	50116	44	5.80	5	30	35.57523	.01149	-1	45	5.1564	.0514	-.152025	0.006154	-0.75328	0.01424
1517	50127	75	7.86	5	30	37.22925	.00564	-1	57	36.5350	.0233	0.010531	0.002757	0.29601	0.01040
1518	0	31	10.95	5	32	32.78168	.00940	-0	57	37.8466	.0285	-.003202	0.001440	-0.09614	0.00934
1519	0	41	10.60	5	30	38.73266	.00725	-1	37	30.6056	.0266	-.013294	0.002180	0.04914	0.01137
1520	50144	87	7.90	5	30	43.67596	.00527	-1	45	14.6589	.0170	0.057805	0.003878	-0.14227	0.01053
1521	50174	106	8.90	5	30	52.70944	.00373	-1	5	32.9629	.0147	0.013364	0.001984	0.34883	0.00812
1522	0	42	11.00	5	30	54.00985	.01014	-1	30	14.1819	.0356	0.015296	0.005960	-0.22472	0.01738
1523	0	32	10.90	5	30	55.52250	.00637	-1	5	49.3187	.0226	0.003678	0.003358	0.21378	0.00902
1524	0	46	9.50	5	30	56.55385	.00598	-1	32	55.6270	.0295	-.004414	0.002879	0.05368	0.01221
1525	50202	43	4.75	5	30	59.05093	.02633	-1	11	22.8141	.0741	-.060000	0.009850	-0.75917	0.02718
1526	0	37	10.70	5	31	2.06529	.00679	-1	14	47.3211	.0203	0.082687	0.003735	-2.57529	0.00912
1527	0	33	10.65	5	31	3.96700	.00778	-1	46	38.4719	.0288	0.007759	0.003366	-0.57117	0.01334
1528	0	57	8.50	5	31	6.34767	.00651	-1	37	27.4016	.0330	0.007642	0.003326	0.09418	0.01384
1529	0	37	10.60	5	31	9.33341	.00956	-1	13	36.9386	.0337	0.033009	0.004281	-0.57524	0.01064
1530	0	72	8.70	5	31	9.62490	.00626	-1	27	28.8711	.0260	-.064184	0.002787	-0.68083	0.01113
1531	0	33	11.15	5	31	9.81533	.01028	-1	27	59.9021	.0393	-.096632	0.005800	-0.69209	0.01646
1532	0	34	10.50	5	31	13.20535	.01511	-1	42	24.2094	.0388	0.059996	0.005919	-2.63163	0.01295

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
1533	50249	81	8.50	5 31 17.34684	.00587	-1 45 15.2799	.0204	0.060724	0.003497	-2.44070	0.01537
1534	0	48	9.35	5 31 18.12917	.00601	-1 35 7.5832	.0234	-.000793	0.002271	0.08874	0.00457
1535	0	39	11.15	5 31 29.53770	.00850	-1 4 9.9837	.0311	0.017667	0.002668	0.17857	0.01304
1536	50288	93	6.75	5 31 28.77135	.00561	-1 47 14.8692	.0204	0.008580	0.005562	-0.09625	0.01798
1537	0	41	10.05	5 31 28.99720	.00743	-1 48 51.9664	.0381	0.068073	0.001116	-2.52957	0.00884
1538	50298	74	5.65	5 31 31.34984	.00770	-1 4 7.4532	.0383	0.003628	0.004034	-0.25733	0.01725
1539	0	57	9.05	5 31 29.64336	.00951	-1 45 14.0526	.0315	0.025644	0.004133	-0.03361	0.01462
1540	50301	51	7.60	5 31 32.03597	.01149	-1 30 10.8104	.0449	-.118093	0.005364	-2.41599	0.01657
1541	0	39	10.95	5 31 37.21380	.00912	-1 3 1.8425	.0437	0.005712	0.003836	0.66661	0.01665
1542	50318	70	6.70	5 31 38.21226	.00889	-1 56 4.9991	.0280	0.008998	0.007129	0.64326	0.01916
1543	514694	45	10.03	5 31 44.23487	.00885	-1 54 49.9266	.0358	0.006542	0.005101	0.37034	0.01855
1544	0	32	10.27	5 31 47.07833	.01292	-2 1 40.2890	.0592	0.013608	0.004890	0.26418	0.02153
1545	514697	73	8.73	5 31 48.70671	.00770	-1 53 2.1503	.0249	0.023415	0.003207	-0.80660	0.01220
1546	0	36	10.90	5 32 3.53095	.00596	-1 2 50.4768	.0220	-.010208	0.003822	0.32806	0.00828
1547	0	42	10.90	5 32 3.70193	.00682	-1 2 58.5254	.0201	-.004973	0.003705	0.19426	0.01241
1548	50398	90	8.35	5 32 2.65251	.00551	-1 46 33.4199	.0217	-.065124	0.003085	-0.85911	0.01179
1549	0	32	11.00	5 32 4.02314	.01507	-1 35 52.9824	.0320	-.036717	0.006241	-0.71846	0.01058
1550	0	40	9.33	5 32 8.39678	.01434	-1 59 24.2969	.0616	-.002254	0.004895	0.28549	0.02216
1551	0	88	9.90	5 32 11.05013	.00399	-1 2 8.4090	.0127	-.022727	0.001688	-0.00536	0.00558
1552	0	51	10.00	5 32 16.17819	.00889	-1 4 23.1910	.0266	-.007508	0.003006	0.22822	0.01072
1553	0	33	11.00	5 32 17.29480	.00829	-1 8 22.9271	.0311	0.019492	0.004452	0.33347	0.01818
1554	0	51	10.90	5 32 17.30564	.01479	-1 10 34.8372	.0274	-.201775	0.005118	-1.65548	0.01431
1555	0	39	10.80	5 32 25.08372	.00966	-1 30 7.1647	.0324	-.011915	0.004008	1.89113	0.01342
1556	0	33	10.85	5 32 26.26460	.01095	-1 8 3.4361	.0285	-.015415	0.006092	-1.48415	0.01565
1557	0	49	10.65	5 32 27.52105	.00672	-1 29 10.0620	.0470	0.079664	0.001817	-1.67386	0.01805
1558	0	35	10.10	5 32 26.29859	.01466	-1 56 17.0609	.0475	-.046636	0.004647	-0.01008	0.01691
1559	0	87	8.20	5 32 32.44426	.00824	-1 26 0.9982	.0317	-.046753	0.003246	0.57448	0.01232
1560	0	31	10.95	5 32 36.42767	.01321	-1 52 13.3719	.0537	-.028332	0.005051	0.40563	0.01911
1561	0	66	8.55	5 32 41.38188	.00634	-1 42 39.7246	.0323	-.011714	0.002060	-0.08257	0.01029
1562	50542	64	8.20	5 32 43.65826	.00949	-1 56 0.8242	.0497	-.027715	0.004094	-0.77438	0.01971
1563	50567	84	8.30	5 32 48.76817	.00607	-1 3 54.0338	.0255	-.011163	0.002564	-0.15404	0.01077
1564	0	29	10.33	5 32 50.79347	.01398	-1 55 14.5819	.0653	-.023500	0.005843	-0.65418	0.02482
1565	0	81	8.75	5 32 53.92755	.00483	-1 21 31.2837	.0233	-.047962	0.002305	-0.00771	0.00874

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
1566	0	70	8.85	5 32 55.02705	.00516	-1 4 57.9480	.0183	-.014023	0.002865	-0.05914	0.00868
1567	0	33	10.05	5 32 54.17010	.01148	-1 36 22.5303	.0376	-.099430	0.004364	-0.00847	0.01461
1568	50608	60	8.50	5 32 58.61795	.00845	-1 37 45.3391	.0347	-.141450	0.004023	0.35728	0.01375
1569	50614	50	8.10	5 33 1.28295	.00981	-1 52 38.3427	.0322	0.024205	0.004731	-0.02128	0.01558
1570	0	31	10.20	5 33 4.49085	.01054	-1 36 24.7783	.0325	-.016832	0.003989	0.27948	0.01263
1571	50628	51	8.35	5 33 4.16157	.01057	-1 48 43.6016	.0460	-.005811	0.004315	-0.12527	0.01952
1572	0	32	10.95	5 33 12.97658	.01079	-1 28 26.7184	.0401	-.048879	0.003320	-0.57804	0.01446
1573	0	34	9.87	5 33 13.51984	.01241	-1 57 55.4344	.0524	-.039162	0.003919	0.30121	0.01538
1574	0	23	11.00	5 33 16.39863	.01489	-1 33 44.1183	.0434	-.023766	0.005800	-0.35894	0.01686
1575	0	37	10.75	5 33 17.38357	.00739	-1 16 0.6749	.0326	-.030191	0.001321	0.37375	0.01556
1576	0	27	10.80	5 33 18.66341	.00884	-1 40 2.2805	.0302	-.053152	0.003359	-2.12541	0.00961
1577	0	25	10.85	5 33 19.60490	.01192	-1 44 12.4603	.0406	-.101121	0.004315	2.32468	0.01519
1578	0	33	10.85	5 33 20.80351	.00757	-1 27 11.7520	.0389	-.046689	0.002527	-0.40165	0.01439
1579	0	14	9.95	5 33 26.79015	.02804	-1 10 24.9553	.1244	-.242964	0.008774	0.14746	0.06215
1580	0	64	8.40	5 33 27.89674	.00768	-1 25 43.4380	.0222	0.046704	0.002957	2.38443	0.00903
1581	0	42	8.77	5 33 26.98530	.00976	-1 53 36.8408	.0352	-.014460	0.002670	0.04396	0.01027
1582	50710	67	8.25	5 33 27.30632	.01050	-1 47 36.8845	.0409	-.011630	0.004694	-0.19134	0.01812
1583	0	30	10.15	5 33 30.60605	.00854	-1 45 55.1062	.0389	-.015624	0.003334	-0.24004	0.01446
1584	0	39	9.90	5 33 31.13437	.00772	-1 33 58.9850	.0511	-.017226	0.001643	0.34682	0.01924
1585	0	27	10.85	5 33 38.07662	.01194	-1 42 12.1293	.0315	-.024831	0.004320	0.04789	0.01308
1587	0	44	8.60	5 33 44.41422	.00887	-1 33 0.9886	.0316	-.014689	0.003431	0.38210	0.01305
1588	50782	60	7.80	5 33 46.00834	.00894	-1 39 56.0436	.0493	-.009580	0.004194	0.04728	0.01879
1589	0	19	10.77	5 33 45.80144	.01402	-2 1 13.6548	.0587	-.016461	0.002786	-0.11890	0.01640
1590	0	28	10.15	5 33 46.79116	.01614	-1 52 38.6086	.0477	-.029149	0.005912	0.27310	0.01499
1591	0	22	10.13	5 33 55.34693	.01345	-1 57 51.6850	.0471	-.035703	0.004198	1.11927	0.01163
1592	50817	37	10.20	5 33 56.40375	.00641	-1 47 3.7019	.0383	-.107548	0.004402	0.39396	0.01498
1593	514813	140	7.95	5 33 59.26695	.00516	-1 15 59.9035	.0272	0.038150	0.002542	-0.30356	0.01037
1594	50825	41	7.40	5 33 59.15379	.01575	-2 0 49.6514	.0539	0.018141	0.005651	-0.39058	0.01943
1595	0	102	9.95	5 34 1.73328	.00631	-1 11 4.0000	.0265	0.019486	0.002513	0.73840	0.01039
1596	50854	132	7.35	5 34 4.53242	.00489	-1 3 28.1729	.0252	-.000673	0.002028	-0.45429	0.00972
1597	0	48	8.70	5 34 4.00707	.01180	-1 59 4.4738	.0661	-.049945	0.003458	0.09336	0.02136
1598	0	52	10.45	5 34 13.21176	.00830	-1 8 37.8743	.0371	0.110168	0.002558	-2.87477	0.01532
1599	0	99	8.30	5 34 18.65407	.00596	-1 26 54.5429	.0356	-.020191	0.002297	0.32376	0.01337

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
1600	0	48	10.60	5 34 22.27596	.01185	-1 16 50.4718	.0462	0.019977	0.004418	2.88827	0.01674
1601	0	80	8.63	5 34 24.49975	.00560	-1 20 12.9170	.0187	-.024417	0.002012	0.58086	0.00754
1602	0	82	8.23	5 34 25.25938	.00756	-1 27 53.7166	.0299	-.023077	0.002992	0.74261	0.01134
1603	0	34	9.50	5 34 27.16378	.00847	-1 34 18.0646	.0227	-.068114	0.003258	0.72330	0.00762
1604	0	15	10.57	5 34 27.03417	.03259	-1 57 40.7835	.0816	-.121423	0.006456	-0.87732	0.02473
1605	50932	53	7.90	5 34 30.60558	.00556	-1 39 7.1997	.0249	-.036686	0.002545	0.65617	0.00960
1606	0	44	11.05	5 34 33.52083	.01220	-1 14 40.6565	.0512	0.022053	0.004668	1.17826	0.01920
1607	50974	78	7.20	5 34 42.72789	.00732	-1 41 48.7713	.0172	-.038454	0.002958	0.85887	0.00735
1608	50998	66	7.95	5 34 48.74905	.00978	-1 48 1.1488	.0468	-.096771	0.005153	1.03002	0.02156
1609	0	36	11.05	5 34 57.08941	.01081	-1 4 42.5526	.0468	0.010818	0.004940	0.26601	0.01851
1610	0	41	10.40	5 34 57.61643	.00776	-1 5 9.5985	.0265	-.047233	0.002941	1.82571	0.00997
1611	51045	116	6.45	5 35 2.72075	.00627	-1 27 3.1497	.0342	-.033177	0.002519	0.95361	0.01409
1612	0	35	10.50	5 35 6.80380	.01266	-1 53 5.6523	.0415	-.054055	0.004647	-0.76445	0.01599
1613	51079	121	7.80	5 35 11.66922	.00429	-1 19 4.2529	.0212	-.024818	0.001886	0.70426	0.00842
1614	51099	58	7.85	5 35 17.27545	.01076	-1 36 51.7200	.0448	-.072500	0.004493	0.58093	0.01616
1615	0	48	10.75	5 35 20.39142	.00683	-1 29 19.3019	.0430	-.115253	0.002511	-0.06378	0.01612
1616	51147	62	8.50	5 35 32.99340	.01055	-1 17 3.0206	.0439	-.058125	0.003974	0.94684	0.01650
1617	0	40	9.57	5 35 33.41822	.01102	-1 53 13.2688	.0422	0.014454	0.004415	0.26927	0.01449
1618	51160	69	7.60	5 35 36.31133	.01044	-1 46 48.7337	.0314	-.059160	0.004177	0.31765	0.01228
1619	51170	44	8.70	5 35 38.35800	.01031	-1 4 41.6844	.0602	0.008033	0.003769	0.29749	0.02223
1620	51183	112	6.20	5 35 41.35169	.01473	-1 11 49.6790	.0812	-.030921	0.005504	0.18614	0.03012
1621	0	67	9.95	5 35 49.48727	.00668	-1 23 31.1280	.0251	-.013667	0.002690	0.05314	0.00899
1622	0	44	10.65	5 35 50.99005	.01289	-1 10 0.1472	.0544	0.017961	0.003788	1.71551	0.01947
1623	51248	65	7.40	5 36 0.20912	.01224	-1 5 9.9292	.0533	-.204621	0.004673	0.59239	0.02015
1624	0	7	11.10	5 35 59.87429	.02382	-1 55 42.2231	.0664	-.038118	0.008632	0.14135	0.02011
1625	0	35	10.50	5 36 9.29161	.00952	-1 25 28.8655	.0321	-.024692	0.003223	-0.11095	0.00939
1626	514926	33	10.45	5 36 17.91274	.01639	-1 11 59.8314	.0730	-.137013	0.005650	-0.05916	0.02566
1627	51327	103	8.25	5 36 22.84251	.01071	-1 11 49.5301	.0552	0.233245	0.004176	-1.70083	0.02058
1628	51362	80	8.35	5 36 32.44738	.01073	-1 15 27.8395	.0463	-.179073	0.004052	0.36534	0.01747
1629	0	48	9.45	5 36 33.26768	.00936	-1 33 33.0435	.0212	-.057394	0.003492	-0.38037	0.00617
1630	51367	24	7.93	5 36 34.87016	.01735	-2 1 43.6775	.0717	0.000294	0.005642	0.12676	0.02506
1631	0	22	10.75	5 36 39.59762	.01381	-1 1 44.5319	.0363	-.007126	0.004551	-0.26738	0.00971
1632	0	11	10.63	5 36 43.47103	.02277	-1 55 29.4278	.0743	-.022337	0.006213	0.01531	0.02399

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
1633	0	41	10.80	5 36 46.96624	.01063	-1 22 5.7046	.0273	-.053803	0.004149	0.00693	0.00884
1634	0	41	10.05	5 36 47.14247	.00902	-1 18 4.0005	.0407	-.030914	0.003559	-0.96845	0.01523
1635	0	26	10.65	5 36 49.20444	.00611	-1 12 59.6584	.0477	-.010095	0.001624	-0.05243	0.01661
1636	0	47	9.65	5 36 51.40508	.01419	-1 9 38.1232	.0440	0.081214	0.004475	-2.53519	0.00791
1637	0	21	11.05	5 36 52.13286	.02188	-1 13 21.6933	.0752	-.048634	0.007934	-0.21674	0.01411
1638	0	61	8.40	5 36 53.03966	.00785	-1 37 14.9597	.0262	-.045900	0.003146	-0.06420	0.01026
1639	0	37	10.35	5 36 55.50156	.01093	-1 11 46.3864	.0504	-.026456	0.003911	0.11334	0.01815
1640	0	35	11.05	5 37 4.09153	.01191	-1 27 17.6822	.0361	-.003501	0.004537	0.36816	0.01486
1641	0	85	8.40	5 37 5.27612	.00628	-1 24 14.0823	.0212	-.040056	0.002424	0.12371	0.00862
1642	0	16	9.63	5 37 23.43432	.01967	-1 57 47.6239	.0582	-.024118	0.006255	0.28660	0.01056
1643	51524	63	9.10	5 37 25.13800	.00905	-1 31 34.8721	.0420	0.009037	0.004452	-1.03560	0.01799
1644	51519	24	6.87	5 37 25.05615	.02593	-1 57 9.0862	.0626	-.023633	0.008410	0.13325	0.01935
1645	0	41	10.65	5 37 31.76285	.00811	-1 21 15.3828	.0414	-.117131	0.003126	-0.33731	0.01553
1646	51551	97	7.25	5 37 31.89791	.00860	-1 27 4.2784	.0425	-.016060	0.003482	0.14699	0.01688
1647	51546	78	8.10	5 37 31.57950	.00653	-1 44 18.4965	.0256	-.095122	0.003301	-0.82649	0.01129
1648	51565	82	8.30	5 37 34.83652	.00864	-1 41 22.3312	.0395	0.192608	0.003859	-11.16849	0.01573
1649	0	39	9.00	5 37 36.88996	.01274	-1 13 11.1437	.0337	-.083399	0.004583	-0.02602	0.01132
1650	0	22	11.00	5 37 41.08499	.01056	-1 10 19.4485	.0629	-.009976	0.002403	-0.47252	0.02181
1651	51588	93	7.15	5 37 41.49167	.00825	-1 29 17.0523	.0401	-.062140	0.003232	0.03747	0.01501
1652	51599	65	9.10	5 37 44.21350	.00935	-1 32 17.9033	.0383	-.072874	0.004074	-4.21067	0.01711
1653	0	15	11.00	5 37 54.22073	.01327	-1 19 24.2744	.0735	-.205158	0.004130	-0.91396	0.02595
1654	0	20	10.60	5 37 54.86306	.01764	-1 48 21.6920	.0541	0.018702	0.007563	-0.47233	0.02266
1655	0	40	9.05	5 37 56.53656	.00961	-1 25 25.1243	.0299	0.008723	0.003869	-1.75778	0.01166
1656	0	15	10.05	5 38 10.08840	.01273	-1 12 6.0362	.0577	-.257733	0.004048	-0.94543	0.01917
1657	0	17	8.38	5 38 14.72039	.02673	-1 57 5.7411	.0815	0.037803	0.006600	-0.12759	0.02482
1658	51704	17	5.13	5 38 18.22872	.02872	-1 9 12.9477	.1309	-.116756	0.009091	-0.50270	0.04543
1659	51720	52	7.07	5 38 24.35526	.01066	-1 31 54.3083	.0402	-.032007	0.004245	-0.53557	0.01669
1660	51736	44	8.30	5 38 27.51170	.01854	-1 11 0.7107	.0501	-.069596	0.006649	-0.53283	0.01790
1661	0	34	9.03	5 38 25.90031	.01300	-1 51 44.4766	.0451	0.007449	0.003574	1.02724	0.01973
1662	51797	16	8.33	5 38 44.97703	.01610	-1 3 0.7406	.0582	-.054627	0.004408	-2.75233	0.01966
1663	51804	32	8.03	5 38 47.80626	.01101	-1 21 2.4663	.0519	-.084197	0.003675	-0.15456	0.01845
1680	0	10	11.05	5 38 52.65158	.04125	-1 51 2.1249	.1337	-.061844	0.007952	-0.28610	0.03045
1681	51862	32	9.40	5 39 4.55316	.02585	-1 42 45.4204	.1247	0.020573	0.008685	-9.03974	0.04444

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
1682	0	8	11.20	5 39 10.35178	.03445	-1 49 32.7707	.1260	-.072962	0.005024	-1.49316	0.03077
1684	0	10	10.95	5 40 33.04406	.01725	-1 34 46.8513	.0805	-.131961	0.009395	-0.32363	0.03971
1685	52181	21	7.85	5 40 37.44956	.01532	-1 38 5.7537	.0683	0.015153	0.006023	-2.74789	0.02068
1686	0	13	10.95	5 41 8.51401	.01405	-1 26 46.1057	.0669	-.006772	0.007955	-1.00975	0.03062
1688	0	11	10.50	5 41 25.12092	.01052	-1 26 16.6879	.0650	0.009720	0.004616	-3.23671	0.01658
1689	0	12	10.90	5 41 24.96556	.01302	-1 38 11.7621	.0385	-.125787	0.006134	-1.11117	0.01922
1690	52469	39	8.25	5 42 1.37506	.00570	-1 38 5.3809	.0232	0.159588	0.004644	-2.96824	0.01411
1691	52470	35	8.75	5 42 3.27021	.00691	-1 40 59.7279	.0235	0.033818	0.004982	1.27762	0.01410
1692	0	5	11.05	5 42 6.97655	.01624	-2 1 7.7263	.0863	0.108499	0.005589	-2.79426	0.03005
1693	0	32	8.13	5 42 14.49636	.00695	-1 28 22.0037	.0313	0.283510	0.004595	1.14905	0.01778
1694	0	9	10.85	5 42 28.46474	.00875	-1 38 2.3515	.0382	-.229355	0.003880	-4.09389	0.02020
1695	52661	41	7.70	5 42 56.61968	.00876	-1 38 1.3402	.0264	0.856706	0.008616	-5.70499	0.02255
1696	52650	39	8.47	5 42 55.96261	.00780	-1 55 38.2604	.0270	-.148391	0.004677	-1.48993	0.01731
1697	0	21	9.60	5 43 1.04108	.01130	-1 28 48.7094	.0433	-.042777	0.004834	-0.66251	0.02921
1698	0	11	10.40	5 43 40.16661	.01284	-1 41 11.3175	.0596	-.103549	0.005836	-1.73536	0.01469
1699	52812	35	8.25	5 43 40.48660	.00787	-1 32 30.3547	.0237	-.034505	0.004955	0.38737	0.01613
1700	0	19	9.80	5 43 42.66996	.01228	-1 31 52.8119	.0455	-.060451	0.006001	0.08051	0.01654
1702	0	3	10.95	5 44 13.25745	.06281	-1 35 31.1902	.1986	-.128256	0.009645	-0.02011	0.02838
1703	52944	11	8.50	5 44 22.10341	.02224	-1 41 57.1804	.0878	-.063086	0.007198	0.93690	0.02897
1704	52946	8	8.60	5 44 23.05029	.03484	-1 44 12.9459	.0779	-.108084	0.011572	-0.51186	0.01286
1705	0	6	9.85	5 44 31.76164	.03863	-1 31 11.9174	.1431	-.070779	0.007193	-0.45150	0.03710
1789	0	17	10.80	5 14 32.20947	.01415	-2 23 17.5169	.0482	0.014226	0.006551	-3.29478	0.02339
1790	47240	62	7.65	5 14 40.17314	.00476	-2 0 56.8665	.0147	0.038505	0.003299	0.07247	0.00962
1791	47237	10	8.25	5 14 39.05755	.04916	-2 37 58.8428	.1226	-.145201	0.011678	1.21912	0.02634
1792	0	17	9.20	5 14 44.55478	.01268	-2 16 14.9842	.0562	-.062776	0.007610	2.89751	0.05032
1793	0	7	10.75	5 14 44.43893	.05002	-2 30 49.7029	.2326	-.055085	0.008231	-0.73116	0.04179
1794	0	24	9.45	5 14 47.67654	.01218	-2 12 13.5087	.0468	-.070079	0.009312	-0.27244	0.03974
1795	0	7	11.10	5 14 47.98073	.05861	-2 33 22.8488	.1679	-.117762	0.010556	-0.98919	0.04028
1796	0	7	10.40	5 14 57.11442	.02967	-2 27 48.3745	.1128	-.040793	0.006077	-0.80766	0.02992
1798	0	23	9.75	5 15 12.51040	.01231	-2 7 41.0763	.0384	-.209620	0.012170	1.87413	0.03563
1799	0	7	10.40	5 15 14.83018	.03797	-2 39 47.4065	.1079	-.077418	0.006782	-1.77177	0.02343
1800	0	37	9.35	5 15 25.08451	.00577	-2 0 54.2104	.0187	-.097703	0.001867	-2.25262	0.00949
1802	47364	39	9.20	5 15 24.77894	.00836	-2 19 10.5115	.0322	-.220503	0.008075	0.47339	0.02830

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
1803	0	19	10.35	5 15 25.18300	.01031	-2 14 58.7206	.0452	-.098159	0.008482	0.16808	0.03587
1804	47452	13	8.15	5 15 53.54030	.03366	-2 35 14.2976	.1310	-.059770	0.005453	-0.38047	0.03511
1805	0	13	10.40	5 15 58.33546	.01262	-2 17 37.3970	.0512	-.089560	0.010043	0.05627	0.02528
1807	0	15	10.60	5 16 5.61596	.01319	-2 16 38.7635	.0507	-.181728	0.009294	0.56339	0.03439
1808	47501	61	8.60	5 16 7.56565	.00481	-2 5 23.8124	.0165	-.001667	0.002807	-1.86041	0.01237
1809	0	29	9.00	5 16 7.06359	.00891	-2 22 48.7230	.0326	-.086579	0.005416	-2.18027	0.02256
1810	0	30	9.55	5 16 9.67017	.01198	-2 9 14.5611	.0426	-.101462	0.011729	0.04877	0.04171
1811	0	11	11.20	5 16 17.95727	.01448	-2 22 15.4220	.0573	-.047609	0.002455	-1.01087	0.04576
1812	0	34	8.65	5 16 21.79229	.01252	-2 9 19.3905	.0339	-.098160	0.011275	0.16905	0.02762
1814	0	3	11.35	5 16 29.16998	.09030	-2 51 9.3229	.2609	-.073693	0.018515	-3.62330	0.03719
1815	0	19	10.05	5 16 37.80431	.01121	-2 32 49.8069	.0388	-.017293	0.003677	-0.90770	0.02787
1816	0	13	11.40	5 16 40.03296	.01526	-2 45 24.5710	.0726	-.059437	0.006620	-1.26438	0.06140
1817	0	6	11.35	5 16 41.15852	.01629	-2 51 9.1861	.1066	0.189437	0.007691	-1.46917	0.07314
1818	0	4	11.15	5 16 42.22324	.03972	-2 28 32.2110	.1403	-.029035	0.013697	-8.09882	0.05258
1819	0	13	11.00	5 16 47.04644	.01965	-2 42 0.3352	.0568	-.068676	0.012030	-1.21196	0.04352
1820	0	9	11.15	5 16 55.96116	.03335	-2 35 6.5139	.0789	-.077531	0.016162	-0.65369	0.07323
1821	47656	87	8.40	5 17 2.39160	.00911	-2 22 21.6090	.0377	-.000503	0.009470	-1.10474	0.04002
1822	0	21	11.05	5 17 14.00541	.01616	-2 9 4.0298	.0694	-.123691	0.011697	1.52941	0.05058
1823	0	13	11.30	5 17 22.21111	.01681	-2 43 5.6069	.0519	-.089210	0.010148	-0.96303	0.03195
1824	0	21	10.60	5 17 30.18618	.01470	-2 4 22.9165	.0486	-.053714	0.010495	-0.70920	0.03802
1826	0	11	11.15	5 17 31.62086	.02836	-2 32 9.1382	.1461	-.052983	0.012460	0.99321	0.04694
1827	47739	33	8.50	5 17 31.87936	.01054	-2 31 18.6155	.0577	-.034185	0.010017	-3.10175	0.06131
1828	0	21	10.95	5 17 41.03188	.01225	-2 3 40.4999	.0403	-.077046	0.007542	0.32508	0.00938
1829	0	13	10.95	5 17 41.28411	.01233	-2 36 56.5382	.0422	-.073333	0.006479	-0.19893	0.01260
1830	0	32	8.85	5 17 49.38547	.00599	-2 43 36.8251	.0349	0.059091	0.003632	-1.11400	0.03147
1831	0	13	11.10	5 17 50.71200	.01554	-2 33 51.7340	.0473	-.027110	0.013501	0.04711	0.01693
1832	0	21	10.50	5 17 58.98087	.00895	-2 1 9.1857	.0380	-.036745	0.004799	-0.90427	0.00784
1833	0	12	10.35	5 18 6.22187	.01283	-2 14 11.7936	.0631	-.145307	0.007212	-0.05381	0.04835
1834	0	19	10.90	5 18 35.22361	.01245	-2 26 1.8320	.0508	-.025799	0.005900	-1.67472	0.03877
1835	0	21	11.07	5 18 35.86481	.00971	-2 19 57.6334	.0514	-.054252	0.003740	-0.71770	0.01611
1836	0	23	11.10	5 18 36.49778	.01228	-2 18 6.6021	.0455	-.198555	0.008883	0.40423	0.03009
1837	47953	9	7.85	5 18 41.11067	.04696	-2 54 30.4008	.1441	-.111136	0.008443	-1.21471	0.03031
1838	0	28	11.30	5 24 20.22704	.01161	-1 58 5.1790	.0412	0.033497	0.007872	1.27905	0.02376

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
1839	0	21	11.25	5 18 51.60520	.01075	-2 4 34.5730	.0366	0.074382	0.005751	-0.64785	0.01653
1840	47988	35	9.05	5 18 51.70803	.00880	-2 42 50.5343	.0406	-.067332	0.007744	-0.34153	0.03880
1841	0	10	10.77	5 19 7.84122	.03816	-2 54 29.3816	.1150	-.084572	0.008430	0.20533	0.02366
1842	48034	47	8.20	5 19 8.53416	.00958	-2 5 55.7183	.0519	-.004459	0.004885	0.81198	0.02288
1843	0	43	8.30	5 19 8.54277	.01361	-2 5 55.6072	.0696	-.037652	0.005464	-0.21329	0.02696
1844	0	11	10.30	5 19 10.15935	.06706	-3 0 36.4269	.1927	-.008299	0.011112	0.06909	0.03124
1845	0	18	11.10	5 19 10.78542	.01357	-2 46 41.9124	.0437	-.020013	0.009978	1.01995	0.02740
1846	0	19	10.90	5 19 11.33135	.01216	-2 27 29.2683	.0326	-.065127	0.002472	0.63734	0.01020
1847	0	34	11.10	5 19 24.67657	.01219	-2 17 20.9679	.0322	-.067225	0.007357	-0.36132	0.01404
1848	48082	51	6.47	5 19 27.96516	.01194	-3 0 40.4737	.0524	-.111067	0.005320	-0.94220	0.02194
1849	0	51	9.20	5 19 28.59710	.00658	-2 28 23.9975	.0316	-.106545	0.001763	-1.45391	0.03061
1850	0	60	9.00	5 19 31.34353	.00700	-2 39 6.0018	.0207	-.049544	0.004551	0.99049	0.01594
1851	0	29	10.85	5 19 31.99615	.01212	-2 30 20.1057	.0241	-.147029	0.009582	0.52234	0.01944
1852	0	31	10.95	5 19 35.20834	.01253	-2 29 35.3538	.0212	-.074812	0.005382	-1.22393	0.00636
1853	0	32	10.15	5 19 37.63698	.02060	-2 16 4.0354	.0533	0.094131	0.008111	1.72207	0.02569
1854	0	31	10.75	5 19 37.71375	.00887	-2 35 1.1026	.0281	-.082054	0.006725	-0.23835	0.02221
1855	0	20	11.25	5 19 43.91194	.02020	-2 13 51.0700	.0617	0.095338	0.008960	1.63480	0.03327
1856	0	28	11.05	5 19 52.72771	.00925	-2 7 52.6742	.0445	-.069253	0.005035	1.37921	0.03503
1857	0	33	10.90	5 19 53.91040	.00858	-2 38 47.6472	.0345	-.114344	0.002111	0.25594	0.03508
1858	0	37	10.55	5 20 0.53575	.00815	-2 50 24.0459	.0276	-.088609	0.005867	-0.20538	0.01895
1859	0	39	11.05	5 20 1.43607	.00797	-2 2 45.0176	.0310	-.041073	0.003046	-1.05072	0.01492
1860	0	36	10.50	5 20 2.30895	.00973	-2 52 57.2251	.0299	-.065977	0.007996	0.01643	0.02094
1861	0	44	10.90	5 20 2.85142	.00940	-2 20 47.7362	.0337	0.044544	0.006229	2.02850	0.03199
1862	0	34	10.65	5 20 3.14830	.01622	-2 23 42.8333	.0363	-.052890	0.008876	0.55388	0.00633
1863	0	23	10.55	5 20 13.85736	.01566	-2 12 23.9911	.0533	-.132911	0.014543	-1.35723	0.04704
1864	48239	47	8.15	5 20 26.54628	.00715	-2 11 45.4324	.0334	0.014367	0.003932	0.52970	0.02511
1865	0	20	11.15	5 20 26.88749	.03055	-2 32 4.2140	.0519	0.034495	0.012183	-1.27825	0.03545
1866	0	28	10.67	5 20 29.60231	.02236	-2 59 11.0120	.0740	-.197768	0.004355	0.66362	0.02753
1867	0	11	9.05	5 20 30.21620	.02778	-2 23 8.9548	.1265	0.001077	0.008075	3.73561	0.02684
1868	48255	39	8.40	5 20 30.98032	.01010	-2 1 52.4836	.0369	-.014911	0.006596	0.01021	0.02914
1869	48267	72	8.57	5 20 34.57892	.00725	-2 58 45.8785	.0400	-.303618	0.003490	0.69811	0.01595
1870	0	24	10.65	5 20 35.56865	.00903	-2 47 12.4391	.0383	-.130991	0.006799	-2.05575	0.02627
1871	0	18	11.35	5 20 41.03735	.01425	-2 34 15.5613	.0523	-.092595	0.007198	0.22138	0.03293

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
1872	0	25	11.25	5 20 42.45171	.01182	-2 57 21.6148	.0400	-.024969	0.002716	0.42211	0.02568
1873	48307	71	8.40	5 20 50.36317	.01027	-2 19 21.1119	.0339	0.108294	0.010645	-10.62092	0.03474
1874	48310	22	10.20	5 20 50.58394	.01243	-2 2 58.9894	.0331	0.027113	0.008444	-0.90002	0.02425
1875	0	48	9.65	5 21 0.84137	.00853	-2 23 33.2009	.0309	-.027488	0.006972	0.46310	0.02585
1876	48343	67	8.65	5 21 3.67112	.00598	-2 51 2.9683	.0267	-.055066	0.005109	-0.01122	0.02430
1877	0	21	10.75	5 21 5.23467	.01228	-2 36 39.4249	.0421	-.146262	0.008348	-1.77610	0.03155
1878	0	21	10.90	5 21 7.47456	.03053	-2 5 14.8699	.0804	-.168505	0.015517	-1.32172	0.05027
1879	0	20	10.75	5 21 7.52013	.01744	-2 5 14.9023	.0527	-.085891	0.009908	0.19981	0.02671
1880	0	13	10.45	5 21 7.99832	.02118	-2 10 58.4875	.0330	0.154657	0.018454	-1.43439	0.02257
1881	0	22	10.55	5 21 8.39167	.02989	-2 21 8.9998	.0496	0.112267	0.005776	-0.34937	0.03260
1882	0	19	11.15	5 21 13.64517	.01406	-2 47 1.6060	.0508	-.019936	0.011535	-1.64580	0.04476
1883	0	26	11.20	5 21 15.81240	.01167	-2 51 38.2793	.0679	-.122650	0.009291	0.06693	0.05898
1884	0	25	10.85	5 21 26.93446	.01079	-2 15 20.1988	.0314	-.110112	0.006631	-0.61405	0.01679
1885	0	23	11.03	5 21 29.51059	.01290	-2 55 29.6875	.0433	-.056355	0.007924	-0.79018	0.02824
1886	0	24	10.30	5 21 39.11126	.01296	-2 6 4.4934	.0381	-.257398	0.009804	-2.04377	0.02854
1887	0	21	10.20	5 21 39.96784	.01229	-2 34 5.5728	.0339	-.074770	0.007568	-1.18611	0.02262
1888	0	17	11.40	5 21 40.72930	.01394	-2 47 52.4630	.0505	-.011429	0.007313	-0.65019	0.03189
1889	0	22	11.20	5 21 42.19473	.01644	-2 0 42.0954	.0381	-.179566	0.008528	-1.18470	0.02822
1890	0	26	11.43	5 21 42.07379	.01194	-2 56 8.0090	.0366	-.060297	0.004724	-0.44178	0.01900
1891	0	21	10.45	5 21 46.01416	.01481	-2 51 20.2511	.0646	-.065666	0.012801	-1.27333	0.05092
1892	0	25	11.40	5 21 50.43437	.01299	-2 26 56.5833	.0523	-.115865	0.004242	0.12481	0.03900
1893	0	28	10.65	5 21 51.88181	.01163	-2 28 36.8228	.0374	-.094048	0.005248	0.24790	0.02296
1895	0	28	10.30	5 21 58.33495	.01040	-2 7 18.9395	.0321	-.217501	0.007138	-0.94872	0.01427
1896	48504	92	9.30	5 21 58.54720	.00574	-3 0 58.6023	.0201	-.039189	0.003645	-0.26529	0.01334
1897	0	52	9.95	5 22 3.70675	.00928	-2 15 12.9951	.0289	-.104080	0.004038	-2.61253	0.02081
1898	0	28	11.05	5 22 3.58752	.01428	-2 25 17.3855	.0466	-.258210	0.008948	-0.68587	0.03322
1899	0	19	10.40	5 22 4.11512	.01185	-2 36 55.9251	.0350	-.057163	0.006424	0.83194	0.02706
1900	0	11	10.85	5 22 5.42137	.02092	-2 31 4.0301	.0811	0.049312	0.007237	-4.93286	0.02328
1901	48539	39	6.35	5 22 9.52970	.00863	-2 32 31.4592	.0369	-.074439	0.006413	0.62433	0.03218
1902	0	39	11.23	5 22 13.00248	.01118	-2 49 52.2023	.0366	-.066416	0.006405	-0.22477	0.02808
1903	0	31	9.30	5 22 21.80828	.01512	-2 27 31.3533	.0328	-.067444	0.007646	0.34380	0.01694
1904	0	102	9.17	5 22 21.83774	.00576	-2 51 5.4478	.0275	0.113854	0.005431	0.53828	0.02129
1905	0	24	10.37	5 22 22.28716	.01044	-2 41 22.1823	.0398	-.041562	0.005262	0.60537	0.02426

Table 9: (Continued)

AC#	ACRS#	N	m	α			ϵ_α	δ			ϵ_δ	μ_α	$\epsilon_{\mu\alpha}$	μ_δ	$\epsilon_{\mu\delta}$
1906	0	23	10.03	5	22	27.03669	.01343	-2	40	37.2735	.0332	-.035331	0.007432	0.81329	0.02339
1907	0	21	11.00	5	22	29.00191	.01372	-2	27	26.3997	.0394	0.117580	0.007356	0.62884	0.03441
1908	0	21	10.90	5	22	30.88313	.01207	-2	35	15.4867	.0332	-.034126	0.005563	-0.78181	0.02516
1909	48609	81	6.73	5	22	30.71305	.00648	-2	51	33.5888	.0265	-.057227	0.005101	-1.27994	0.01749
1910	0	45	10.85	5	22	32.28368	.00817	-3	0	30.7158	.0260	0.045590	0.004946	-0.79076	0.01821
1911	48631	48	8.07	5	22	36.89723	.00665	-2	29	55.7028	.0343	-.112773	0.005004	-11.11378	0.03363
1912	0	43	10.20	5	22	41.12680	.01107	-2	22	35.2113	.0537	-.069893	0.004161	0.24562	0.03401
1913	0	28	11.35	5	22	49.05484	.01214	-2	33	18.6305	.0356	-.019208	0.010651	-0.09065	0.02909
1914	0	41	9.30	5	22	49.37303	.01049	-2	13	7.3721	.0373	-.265770	0.008685	-1.56494	0.03297
1915	0	66	8.75	5	22	49.90819	.00848	-2	39	14.8233	.0255	0.038346	0.006785	-0.52845	0.02360
1916	0	32	11.15	5	22	58.28439	.00996	-2	3	9.3632	.0387	-.069640	0.006767	-0.96111	0.02649
1917	0	29	10.65	5	22	58.21830	.01360	-2	30	16.6727	.0424	-.167425	0.005408	0.81335	0.03559
1918	0	44	11.00	5	23	1.71953	.01427	-2	51	14.5275	.0372	0.039817	0.010550	-0.32094	0.03103
1919	0	44	10.40	5	23	5.02915	.00977	-3	2	38.0988	.0333	0.010158	0.004978	-0.57741	0.01824
1920	0	35	10.95	5	23	7.88827	.01504	-2	21	47.7516	.0386	0.149647	0.007502	1.81587	0.02417
1921	48719	81	8.15	5	23	11.77702	.00986	-2	4	59.9297	.0317	-.185875	0.009440	-0.71630	0.02605
1922	0	38	9.95	5	23	12.49467	.01113	-2	13	58.8068	.0374	0.042338	0.009786	0.99523	0.03272
1923	0	26	11.20	5	23	17.53769	.01218	-2	48	45.4204	.0430	0.015037	0.009297	-0.04523	0.03122
1924	0	9	11.45	5	23	18.87275	.01679	-2	33	29.8460	.0460	-.098328	0.009726	0.03710	0.03829
1925	48756	86	7.20	5	23	24.85706	.00593	-2	22	41.3522	.0207	0.160395	0.006030	3.27455	0.01857
1926	48774	38	10.20	5	23	29.78344	.00947	-3	0	16.1116	.0349	0.016518	0.005958	0.22378	0.02183
1927	0	13	11.45	5	23	30.64819	.03673	-2	3	12.1307	.1734	0.020732	0.016614	-3.49271	0.03036
1928	0	23	11.50	5	23	36.94337	.00940	-2	49	38.1023	.0451	-.088789	0.003778	0.21039	0.02529
1929	48807	70	8.40	5	23	37.84085	.00776	-2	18	51.6935	.0281	-.223312	0.007636	-0.30878	0.02317
1930	0	55	9.55	5	23	37.88658	.01184	-2	18	51.1547	.0437	0.118495	0.008975	3.81008	0.03421
1931	0	32	10.35	5	23	38.88566	.01144	-2	20	15.4949	.0367	0.201680	0.009608	0.05293	0.02828
1932	0	11	11.50	5	23	39.18436	.02702	-2	4	52.4509	.0670	-.074387	0.010269	-0.46054	0.02205
1933	0	15	11.40	5	23	40.67251	.02370	-2	5	9.7047	.0801	0.057869	0.012397	0.77563	0.05699
1934	0	22	11.30	5	23	43.11082	.01472	-2	5	20.7104	.0473	0.123330	0.010577	0.34864	0.01942
1935	0	16	10.80	5	23	43.21229	.01181	-2	30	40.3026	.0438	-.053828	0.007068	-0.98764	0.01092
1936	0	26	10.63	5	23	45.03520	.02363	-2	56	28.7382	.0445	-.075500	0.008944	-1.10406	0.02822
1937	0	20	10.85	5	23	51.23353	.01374	-2	12	30.4065	.0636	0.121906	0.008258	0.96682	0.04302
1938	0	55	9.93	5	23	58.37279	.01073	-2	55	51.6700	.0263	0.168173	0.005462	-1.80147	0.01570

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
1939	0	14	11.45	5 24 1.67888	.01175	-2 30 40.3757	.0521	0.045025	0.007556	-0.39049	0.04839
1940	0	26	11.40	5 24 1.56631	.00893	-2 48 29.6457	.0501	-.024831	0.006845	-1.71275	0.02241
1941	0	35	9.15	5 24 5.70449	.00718	-2 36 13.4826	.0352	0.002195	0.003982	0.04337	0.02846
1942	0	21	10.20	5 24 6.64135	.01252	-2 28 46.9696	.0442	-.049904	0.003210	-0.90953	0.02356
1943	0	38	8.95	5 24 9.40672	.01283	-2 11 39.2600	.0343	0.008619	0.005356	0.70103	0.01995
1944	0	20	10.50	5 24 9.62747	.01316	-2 39 7.8118	.0338	0.038372	0.004468	-1.37724	0.02145
1945	0	61	9.53	5 24 19.55823	.00714	-3 2 56.3701	.0240	0.055295	0.005282	-0.80184	0.01903
1946	0	16	11.30	5 24 26.95695	.01311	-2 42 7.2293	.0398	0.007775	0.005776	-1.78950	0.01816
1947	0	21	11.05	5 24 28.24624	.01617	-2 10 0.1567	.0444	0.051868	0.007907	0.10629	0.02515
1948	48963	41	6.10	5 24 28.17470	.00784	-2 24 7.3539	.0455	0.028841	0.005251	2.50954	0.02403
1949	0	9	10.75	5 24 33.78009	.01660	-2 32 43.8998	.0838	0.247597	0.014837	-1.82393	0.06477
1950	0	56	8.90	5 24 34.15059	.00632	-2 57 47.7624	.0283	0.081627	0.005161	-0.09318	0.01800
1951	0	11	11.35	5 24 36.80649	.01467	-2 16 27.3120	.0766	-.034896	0.004061	-1.78512	0.02777
1952	0	75	8.67	5 24 36.67075	.00526	-2 58 13.4445	.0194	0.117813	0.003638	0.15343	0.01404
1953	49007	46	9.00	5 24 41.62286	.00718	-2 33 41.8879	.0252	-.108739	0.006926	-0.38536	0.02117
1954	0	32	10.17	5 24 42.31298	.01046	-2 54 19.1572	.0453	0.017538	0.006649	-0.50305	0.02589
1955	0	25	10.40	5 24 43.89764	.01193	-2 15 23.1135	.0411	0.012326	0.006340	1.02397	0.02761
1956	0	28	11.10	5 24 45.77686	.01135	-2 22 56.0450	.0390	0.019426	0.006720	0.68082	0.01813
1957	0	49	9.60	5 24 46.18965	.00681	-2 22 52.1932	.0318	0.013113	0.003901	0.52508	0.01896
1958	0	45	8.85	5 24 48.76627	.00816	-2 15 38.7179	.0287	0.085774	0.004210	0.98566	0.01905
1959	0	25	10.35	5 24 49.60395	.01061	-2 4 56.9238	.0381	0.081017	0.006365	0.47297	0.02993
1960	0	9	10.55	5 24 54.22948	.01329	-2 30 3.3467	.0833	-.080706	0.009077	-5.36870	0.06270
1961	0	17	11.15	5 24 55.11718	.02361	-2 8 27.8545	.0664	-.016349	0.007662	0.62978	0.03743
1962	0	24	10.70	5 25 1.33853	.01167	-2 17 35.1717	.0512	-.020948	0.004269	-1.72175	0.03504
1963	0	11	10.93	5 25 3.41389	.02274	-2 57 49.2690	.0857	-.010235	0.008016	-3.52228	0.03483
1964	0	35	10.50	5 25 5.60008	.00726	-2 58 35.2927	.0434	0.153225	0.004343	2.31424	0.02774
1965	0	26	10.90	5 25 6.07124	.01106	-2 7 53.7698	.0387	-.037351	0.004939	-0.97390	0.02554
1966	49122	86	6.50	5 25 14.51901	.00631	-2 11 9.6936	.0223	0.019486	0.005692	-0.49806	0.01867
1967	0	39	11.30	5 25 17.84015	.00897	-2 59 5.4466	.0351	0.092374	0.004261	0.22342	0.01752
1968	0	87	9.85	5 25 24.02787	.00665	-2 19 22.2214	.0240	0.039670	0.005785	-0.95799	0.02064
1969	0	50	10.70	5 25 24.89039	.00987	-2 22 7.2459	.0403	-.062151	0.008002	0.17513	0.03846
1970	49173	59	8.90	5 25 29.60556	.00640	-2 3 38.8979	.0214	-.009201	0.005141	0.62923	0.01639
1971	0	21	11.35	5 25 32.16190	.01410	-2 41 10.5240	.0332	-.117288	0.012180	-3.53866	0.01799

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
1972	49193	89	8.75	5 25 37.55327	.00812	-2 30 45.5342	.0311	-.183835	0.008437	-1.57137	0.03310
1973	49218	93	8.90	5 25 43.90319	.00728	-2 39 47.8014	.0201	0.011726	0.007574	-1.77706	0.02038
1974	0	32	11.00	5 25 50.39005	.00809	-2 1 47.4405	.0238	0.061547	0.004345	0.75412	0.01277
1975	0	34	10.40	5 26 2.93272	.00982	-2 38 17.3924	.0334	-.040025	0.005942	-1.06204	0.02384
1976	49301	81	8.00	5 26 9.79470	.00700	-2 46 23.6560	.0313	-.093007	0.006760	-2.28776	0.03367
1977	0	60	9.86	5 26 22.86349	.00680	-2 59 24.4304	.0255	0.002505	0.004726	0.01163	0.01273
1978	0	25	11.37	5 26 27.07906	.01910	-2 1 52.3595	.0415	0.159765	0.007975	1.12886	0.02499
1979	49365	60	8.53	5 26 30.15848	.00631	-2 2 18.3059	.0268	0.108009	0.004317	-0.34853	0.01384
1980	49364	86	8.17	5 26 30.91815	.00955	-2 48 53.5732	.0263	-.242517	0.009127	1.77880	0.02015
1981	49372	62	7.97	5 26 32.42291	.00785	-2 12 53.5868	.0250	-.128009	0.005646	-0.67683	0.01601
1982	49388	60	8.40	5 26 36.45981	.01286	-2 2 57.3758	.0450	0.125488	0.006699	-0.57859	0.02083
1983	514466	42	9.93	5 26 36.72106	.01915	-2 2 54.3696	.0578	0.005897	0.008006	-4.41715	0.02646
1984	0	45	9.87	5 26 39.32281	.00751	-2 35 17.1303	.0310	-.002686	0.005806	-0.66516	0.02733
1985	49417	76	8.17	5 26 49.81476	.00618	-2 2 50.1162	.0216	0.102078	0.004509	1.08751	0.01382
1986	0	33	10.65	5 32 16.90008	.01620	-1 53 42.5477	.0653	-.081041	0.006321	0.63893	0.02425
1987	0	25	11.35	5 33 1.89080	.01521	-1 54 27.7321	.0677	-.051991	0.004268	-0.53287	0.02582
1988	0	13	11.45	5 33 6.39759	.01902	-1 56 54.8835	.0514	0.016927	0.004144	-0.44809	0.01497
1989	0	19	11.45	5 33 57.46251	.02578	-1 56 22.9161	.0898	-.025749	0.005873	-0.31505	0.02850
1990	0	17	11.40	5 26 52.88562	.01743	-2 55 19.4017	.0700	-.026225	0.008964	0.37790	0.06584
1991	0	26	11.15	5 26 53.29715	.02447	-2 40 39.4328	.0615	0.174738	0.013064	-0.05161	0.03596
1992	0	60	9.65	5 26 58.48549	.00865	-2 17 58.2373	.0251	0.187295	0.007457	1.30596	0.01561
1993	0	25	10.90	5 27 1.03018	.01179	-2 26 57.3951	.0383	0.085529	0.010567	-0.03349	0.03370
1994	0	19	10.70	5 27 5.23369	.01408	-2 53 48.7781	.0513	0.018411	0.008506	-1.82476	0.01723
1995	0	21	10.75	5 27 16.55502	.01506	-2 11 5.8386	.0519	0.022253	0.006036	1.24937	0.02978
1996	0	31	9.40	5 27 19.07595	.01092	-2 5 23.2277	.0394	0.107995	0.006669	0.21071	0.01642
1997	49519	97	8.35	5 27 24.36682	.00541	-2 17 32.6986	.0203	0.240949	0.004673	2.71210	0.01696
1998	0	20	10.85	5 27 31.67157	.01925	-2 11 15.3512	.0651	0.132299	0.010696	1.77188	0.03922
1999	0	14	10.80	5 27 37.25171	.02082	-2 6 45.2864	.0738	0.321049	0.008561	-1.95768	0.01334
2000	0	36	9.05	5 27 47.33053	.00982	-2 4 12.4104	.0450	0.093290	0.003866	0.89285	0.01778
2001	0	58	9.30	5 27 46.65286	.01250	-2 50 2.9503	.0268	-.026083	0.008924	0.40961	0.02330
2002	49606	101	8.15	5 27 49.41363	.00450	-2 20 38.0421	.0174	0.248038	0.003404	-10.25720	0.01377
2003	0	54	8.70	5 28 0.16490	.01213	-2 39 22.0122	.0400	0.193988	0.013151	0.71698	0.04243
2004	0	25	10.00	5 28 4.00969	.01351	-2 6 58.5313	.0549	0.083513	0.006087	-1.76804	0.02565

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon_{\mu\alpha}$	μ_δ	$\epsilon_{\mu\delta}$
2005	0	33	10.80	5 28 16.83151	.01311	-2 24 16.9074	.0451	0.068307	0.008068	1.30935	0.02679
2006	0	67	9.90	5 28 17.70155	.00603	-2 2 2.3824	.0272	0.038857	0.002954	-1.10588	0.01263
2007	49701	93	7.85	5 28 18.28754	.00529	-2 24 11.7476	.0242	0.106948	0.004529	2.34915	0.01523
2008	49720	53	8.40	5 28 25.34389	.00811	-2 44 8.1810	.0236	0.000961	0.008490	-0.61613	0.02071
2009	0	57	9.05	5 28 28.48775	.00672	-2 4 14.3092	.0244	-.003768	0.002091	-0.05226	0.01213
2010	0	19	10.25	5 28 34.06395	.01105	-2 40 42.1292	.0433	-.028785	0.008222	-1.01796	0.03130
2011	0	17	10.85	5 28 37.35505	.01239	-2 36 11.2141	.0576	-.165715	0.011680	-3.95135	0.04636
2012	0	38	10.65	5 28 43.28859	.01599	-2 18 45.3328	.0554	-.129342	0.008534	-1.69227	0.04491
2013	0	53	8.10	5 28 49.79849	.01203	-2 8 4.8263	.0380	0.010905	0.005610	-1.10022	0.01077
2014	49791	68	7.70	5 28 49.84381	.00731	-2 8 5.8526	.0359	-.163899	0.005570	2.96552	0.02252
2015	0	51	10.90	5 28 56.63599	.01211	-2 49 12.6684	.0292	-.100115	0.008431	-1.09168	0.00635
2016	49811	59	8.55	5 28 57.64792	.00726	-2 10 3.7118	.0230	-.022611	0.004761	0.69396	0.01732
2017	0	34	11.30	5 29 5.09643	.01189	-2 31 41.9682	.0346	-.081343	0.007263	-2.37443	0.02801
2018	0	20	10.50	5 29 14.54335	.01529	-2 17 29.1900	.0727	-.005483	0.009365	0.36769	0.04058
2019	0	27	10.70	5 29 19.61652	.01108	-2 1 53.2410	.0403	0.003839	0.005639	0.10760	0.01670
2020	0	25	11.30	5 29 24.35803	.01943	-2 21 10.3647	.0585	-.094434	0.009096	-0.80923	0.03178
2021	0	23	10.45	5 29 33.16682	.01358	-2 27 29.0182	.0447	-.039933	0.007360	-1.66129	0.02721
2022	0	15	11.05	5 29 39.48688	.01854	-2 51 53.8609	.0841	-.031967	0.010580	-0.89606	0.06822
2023	0	20	10.55	5 29 53.49128	.01196	-2 9 58.7724	.0585	0.034074	0.002294	-0.49095	0.02900
2024	0	3	10.20	5 30 1.12820	.13777	-2 29 18.5251	.2937	-.131228	0.022182	-0.88115	0.06603
2025	50036	71	8.03	5 30 10.29639	.00689	-2 4 17.8166	.0175	0.013163	0.004200	0.09875	0.01067
2026	50061	52	7.33	5 30 18.59383	.00784	-2 13 53.3067	.0263	-.087827	0.006297	0.65005	0.01992
2027	0	43	8.83	5 30 36.06144	.01014	-2 42 52.6067	.0316	-.034879	0.005315	-1.18757	0.01527
2028	0	32	9.60	5 30 40.61066	.00951	-2 43 8.3293	.0335	-.031832	0.004399	-0.52743	0.01574
2029	0	26	11.10	5 30 41.73450	.01151	-2 5 15.4364	.0361	-.001362	0.008045	-0.00785	0.02550
2030	0	22	10.90	5 30 42.63483	.00976	-2 28 46.1073	.0300	0.009760	0.006750	-0.48914	0.01457
2031	0	65	8.60	5 30 49.18385	.00753	-2 19 48.5861	.0249	-.021300	0.003538	-0.34640	0.01952
2033	0	35	8.75	5 30 55.65384	.01176	-2 3 29.3082	.0410	0.164466	0.007003	-1.34564	0.02022
2034	0	36	10.47	5 30 55.61923	.00849	-2 58 12.4796	.0346	-.004261	0.004552	-0.51577	0.01654
2035	0	17	10.80	5 30 57.53338	.00899	-2 5 54.5220	.0250	-.002030	0.002636	-0.08995	0.00702
2036	0	23	10.65	5 30 57.11215	.01266	-2 31 33.6753	.0422	-.036411	0.006938	-1.48541	0.02973
2037	0	34	10.70	5 31 11.65252	.00961	-2 53 27.6264	.0325	0.034120	0.005300	-0.05667	0.01762
2038	0	28	9.70	5 31 15.04083	.01308	-2 5 52.5066	.0372	-.064291	0.005263	-0.19689	0.02201

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
2039	0	36	8.90	5 31 18.00211	.01002	-2 28 7.6944	.0352	-.010745	0.005736	-0.02788	0.01003
2040	0	31	10.85	5 31 29.11397	.01019	-2 52 30.3221	.0251	0.136197	0.008114	-4.93641	0.01592
2041	0	39	10.85	5 31 31.39500	.00947	-2 52 23.0141	.0287	-.004404	0.007255	-0.49035	0.00903
2042	0	52	8.80	5 31 32.18645	.00652	-2 42 52.9606	.0211	0.192787	0.004091	0.90148	0.01053
2043	0	20	11.40	5 31 32.64975	.01481	-2 28 52.0152	.0520	-.043349	0.007643	0.59639	0.04443
2044	0	34	9.80	5 31 33.07960	.01139	-2 12 33.1803	.0307	0.012280	0.004312	-0.79153	0.01954
2045	0	25	10.90	5 31 35.53318	.01383	-2 25 4.9286	.0677	-.005003	0.007130	-0.31298	0.02248
2046	0	29	11.50	5 31 36.37731	.01565	-2 0 43.7805	.0547	-.036175	0.006177	0.13803	0.02101
2047	0	23	10.50	5 31 35.90896	.01336	-2 25 46.8948	.0379	-.092210	0.005555	-0.06390	0.02555
2048	0	36	9.30	5 31 36.54015	.01190	-2 16 59.8970	.0432	-.030003	0.003987	0.21051	0.02099
2049	0	23	11.00	5 31 38.90539	.01355	-2 34 14.9196	.0396	-.078388	0.007720	0.14548	0.02109
2050	0	38	9.55	5 31 43.92525	.01250	-2 10 45.3507	.0327	0.025345	0.005250	0.30509	0.02017
2051	0	30	10.05	5 31 44.33902	.01077	-2 33 35.7783	.0481	0.008136	0.006333	-1.11092	0.03902
2052	50335	104	6.85	5 31 44.53147	.00467	-2 54 51.8510	.0219	0.108577	0.003754	1.22377	0.01249
2053	50350	77	7.85	5 31 46.64027	.00710	-2 25 4.7062	.0338	0.019633	0.005153	0.58576	0.01669
2054	0	30	11.10	5 31 48.30444	.01047	-2 27 14.9469	.0421	-.120151	0.003687	-1.99026	0.03116
2055	0	52	10.90	5 31 49.72168	.00896	-2 52 34.7037	.0324	-.010574	0.007320	-0.17084	0.02666
2056	0	38	10.70	5 31 53.17166	.01215	-2 23 14.3812	.0399	0.055985	0.005021	-0.04118	0.01507
2057	0	31	10.35	5 31 53.76250	.00889	-2 43 22.9727	.0321	0.020614	0.007137	-0.26507	0.01374
2058	0	29	10.00	5 31 58.99651	.01924	-2 6 41.3366	.0790	-.074693	0.008541	1.67803	0.02700
2059	0	25	10.60	5 32 4.73004	.02749	-2 9 10.9464	.0838	-.061941	0.009871	-0.26284	0.02946
2060	0	72	9.60	5 32 9.47726	.00742	-2 34 24.3854	.0276	0.137139	0.006223	-0.06316	0.02058
2061	0	82	8.95	5 32 11.86599	.00819	-2 31 0.2874	.0293	0.140404	0.007724	0.14588	0.02565
2062	0	40	10.75	5 32 14.35928	.00995	-2 28 12.0589	.0269	0.012782	0.007992	-0.51877	0.01372
2063	0	50	10.80	5 32 16.47220	.01085	-2 49 19.8550	.0403	-.065346	0.003324	-0.22269	0.02129
2064	0	42	11.45	5 32 16.58044	.00876	-2 48 2.5464	.0337	-.004173	0.004546	0.86682	0.02990
2065	0	45	11.25	5 32 21.55226	.00791	-2 33 32.9769	.0266	0.112851	0.005837	-0.43315	0.02106
2066	0	55	10.15	5 32 22.40423	.01210	-2 22 21.2194	.0357	-.053912	0.005372	-0.32047	0.01786
2067	0	19	10.95	5 32 23.25655	.02255	-2 10 46.7116	.0776	0.017292	0.005082	-0.16653	0.02313
2068	0	41	11.25	5 32 22.74820	.00787	-2 45 21.3948	.0249	0.051135	0.004992	-0.14423	0.01367
2069	0	23	10.10	5 32 32.29118	.02034	-2 14 43.9031	.0901	-.064333	0.003493	0.43989	0.02607
2070	0	22	10.45	5 32 34.14078	.02001	-2 40 37.7435	.0832	0.247633	0.015130	-1.88511	0.07616
2071	0	20	10.25	5 32 39.51410	.01939	-2 14 1.9151	.0946	0.052591	0.003105	-1.18335	0.02672

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
2072	0	32	9.30	5 32 42.97324	.01539	-2 9 13.9359	.0886	-.025156	0.004048	0.21335	0.03133
2073	50532	70	7.80	5 32 42.83507	.00537	-2 24 46.0212	.0245	-.021089	0.004938	-0.18777	0.01598
2074	0	29	11.35	5 32 44.86015	.01093	-2 23 1.1802	.0392	-.013726	0.006578	-1.41097	0.03006
2075	0	13	10.80	5 32 47.58148	.04417	-2 18 48.4974	.2030	0.061248	0.009215	0.79402	0.03621
2076	0	25	11.05	5 32 51.17032	.00936	-2 41 46.5342	.0324	-.053848	0.003084	-0.71765	0.01628
2077	0	19	10.20	5 33 0.04705	.02986	-2 6 8.6292	.1197	-.036033	0.008310	-0.99192	0.03743
2078	0	63	8.65	5 33 5.23727	.00640	-2 29 10.4816	.0225	0.182926	0.006154	-0.54940	0.01585
2079	50677	70	8.30	5 33 15.56378	.00951	-2 24 17.7759	.0289	-.038323	0.004901	-2.41669	0.01767
2080	0	7	11.45	5 33 21.90756	.05225	-2 9 54.7386	.0683	-.049434	0.011778	-0.55919	0.01496
2081	0	60	9.83	5 33 21.79596	.00656	-2 56 12.4719	.0227	0.108841	0.004886	1.05717	0.01639
2082	0	25	10.60	5 33 28.85993	.00871	-2 28 36.7819	.0382	-.068020	0.004056	-0.57789	0.01499
2083	0	23	9.65	5 33 32.53366	.01472	-2 12 8.4940	.0850	0.008152	0.004820	-1.52915	0.02489
2084	50766	29	8.10	5 33 42.97177	.01883	-2 17 21.0740	.0752	0.110258	0.008190	-1.45572	0.02929
2085	0	16	11.50	5 33 55.14886	.01074	-2 44 4.8961	.0281	0.175576	0.005365	-0.25471	0.01423
2086	0	26	11.10	5 34 0.89961	.02867	-2 18 23.7772	.0574	-.027481	0.009116	-0.37880	0.01581
2087	0	22	10.85	5 34 11.78295	.02404	-2 16 45.0757	.1038	-.055784	0.008109	-0.61375	0.03466
2088	0	36	11.20	5 34 11.50979	.00714	-2 38 9.1692	.0264	0.128602	0.004681	-0.39010	0.01379
2089	0	25	8.23	5 34 15.40339	.01441	-2 3 3.5462	.0779	-.018689	0.003440	-0.12672	0.02609
2090	0	27	10.88	5 34 15.69089	.00671	-2 55 54.1105	.0315	0.224076	0.003336	0.39649	0.02021
2091	0	38	8.57	5 34 17.86503	.01237	-2 5 23.8451	.0717	0.027575	0.003788	-0.18454	0.02539
2092	0	15	10.80	5 34 22.72749	.02131	-2 7 35.8313	.0680	0.013690	0.006189	-0.19206	0.02172
2093	0	29	10.38	5 34 22.57169	.01075	-2 59 47.7274	.0360	0.085909	0.004508	0.00434	0.01908
2094	50990	18	10.87	5 34 46.96499	.01226	-2 1 25.1037	.1204	0.091468	0.004910	-0.26361	0.04372
2095	0	43	10.70	5 34 47.96238	.00612	-2 33 20.0881	.0222	0.372511	0.003920	-3.01208	0.01181
2096	0	27	10.65	5 34 50.15526	.01326	-2 51 13.8007	.1126	-.118905	0.006986	1.23689	0.05939
2097	51006	51	7.80	5 34 52.05618	.02684	-2 6 44.7755	.0714	0.038454	0.009701	-2.32249	0.02581
2098	0	127	8.25	5 35 1.31559	.00582	-2 47 2.0482	.0173	0.109670	0.005127	1.12853	0.01197
2099	0	37	9.00	5 35 5.59850	.01754	-2 10 1.0824	.1102	-.031847	0.005645	-0.32978	0.04014
2100	51071	124	8.00	5 35 9.56042	.00378	-2 28 19.7435	.0101	0.231551	0.002330	-0.84590	0.00606
2101	0	43	10.40	5 35 10.92851	.00640	-2 30 50.6102	.0187	-.013855	0.001231	-1.68891	0.00927
2102	0	42	10.80	5 35 14.07143	.00743	-2 31 40.0222	.0230	0.237041	0.003829	-0.18631	0.00857
2103	0	43	10.65	5 35 26.79331	.01295	-2 28 14.5514	.0493	0.347716	0.006450	-3.00600	0.02287
2104	0	42	10.47	5 35 26.95065	.00927	-2 54 59.5354	.0363	0.312214	0.005817	-0.25661	0.02692

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
2105	0	32	11.05	5 35 35.63478	.01013	-2 30 30.1490	.0303	0.207380	0.004025	-1.68477	0.01217
2106	0	46	9.40	5 35 43.00034	.00993	-2 17 40.1115	.0298	0.074463	0.003777	-1.31495	0.01298
2107	0	27	9.90	5 35 47.89500	.02503	-2 6 56.4344	.0739	-.075148	0.008940	0.19292	0.02672
2108	0	33	10.10	5 35 56.96108	.00947	-2 45 12.3107	.0359	0.053953	0.003804	0.77164	0.02338
2109	0	30	11.17	5 35 59.99361	.01166	-2 58 36.1834	.0312	0.326371	0.008037	0.94941	0.01747
2110	0	28	10.47	5 36 1.03402	.01230	-2 56 42.7118	.0346	0.090543	0.006878	0.82676	0.01860
2111	0	27	10.55	5 36 3.58681	.00949	-2 30 26.7110	.0236	0.147309	0.004296	-0.09046	0.00564
2112	0	70	8.35	5 36 3.47413	.00642	-2 35 55.0096	.0269	0.245397	0.004325	-0.51765	0.01339
2113	514913	55	8.05	5 36 3.80504	.02482	-2 35 54.9997	.0452	-.717081	0.009147	-0.31730	0.01931
2114	514914	73	7.45	5 36 5.76820	.00993	-2 34 51.7177	.0197	0.230512	0.005409	-0.12327	0.01022
2115	0	29	10.65	5 36 8.25292	.00922	-2 50 40.2745	.0426	0.032493	0.004414	-0.20605	0.02966
2117	51301	45	6.75	5 36 16.47299	.01472	-2 37 18.5370	.0513	0.193283	0.007720	-0.79179	0.02114
2118	51350	85	7.80	5 36 30.82187	.00443	-2 40 33.5776	.0158	0.237127	0.002592	-0.11215	0.00978
2119	0	63	8.63	5 36 38.89361	.00482	-2 58 11.3916	.0146	0.296376	0.003774	0.70140	0.01002
2120	51395	68	8.30	5 36 44.23972	.00519	-2 33 13.7612	.0144	0.241689	0.002820	-0.18345	0.00770
2121	0	25	10.65	5 36 44.80168	.00817	-2 31 31.5765	.0273	0.160042	0.004010	-4.56142	0.01228
2122	0	45	9.90	5 37 5.92835	.00956	-2 43 51.9712	.0340	0.276964	0.005206	-1.83861	0.01576
2123	0	25	10.70	5 37 9.64776	.00869	-2 27 21.1250	.0237	0.165421	0.004023	-0.48576	0.00849
2124	0	54	8.25	5 37 15.54611	.00717	-2 42 6.1882	.0227	0.030679	0.005313	-0.79353	0.01754
2125	0	25	10.35	5 37 28.32199	.00795	-2 24 27.2641	.0267	0.104321	0.004051	-0.54646	0.00942
2126	0	23	10.65	5 37 36.09201	.00737	-2 30 2.3492	.0290	0.114586	0.004318	-0.38593	0.01146
2127	51584	42	8.40	5 37 42.24860	.00853	-2 32 25.0444	.0201	0.161338	0.007894	-0.65710	0.01448
2128	0	67	9.80	5 37 42.06610	.00619	-2 54 29.0210	.0194	0.181277	0.004078	-2.24224	0.00939
2130	51617	31	6.90	5 37 49.23182	.01689	-2 27 39.5784	.0761	-.084990	0.012253	-0.15700	0.05049
2131	0	26	10.40	5 37 50.47330	.00985	-2 41 56.8070	.0374	0.147651	0.005393	-2.07550	0.02158
2132	0	24	9.40	5 37 56.52582	.01859	-2 27 12.6843	.0791	0.048426	0.012334	-1.86987	0.05202
2133	51668	53	6.15	5 38 6.81379	.00866	-2 51 1.3024	.0380	0.057009	0.006537	0.10737	0.02397
2135	51721	41	9.03	5 38 26.09137	.00718	-2 51 27.7963	.0238	0.188248	0.005155	-1.09622	0.01351
2136	0	14	9.00	5 38 26.95840	.02851	-2 45 45.2730	.0615	0.004156	0.009014	-1.08897	0.01764
2137	51746	9	7.37	5 38 31.62509	.02605	-2 19 44.3827	.0928	-.035245	0.009021	-5.12762	0.03021
2138	51743	18	7.90	5 38 31.73478	.02064	-2 44 29.3441	.0698	0.013742	0.005310	-0.78375	0.02068
2141	0	26	10.33	5 38 46.29032	.00783	-2 58 3.9259	.0339	0.066771	0.003385	0.33094	0.01537
2143	51847	78	8.30	5 39 1.72360	.00493	-2 59 52.9182	.0181	0.115699	0.002467	-0.33986	0.00859

Table 9: (Continued)

AC#	ACRS#	N	m	α	$\epsilon\alpha$	δ	ϵ_δ	$\mu\alpha$	$\epsilon\mu\alpha$	μ_δ	$\epsilon\mu_\delta$
2144	0	7	10.00	5 39 4.84510	.03967	-2 39 43.5245	.1099	-.040757	0.006022	-0.66953	0.02277
2145	51869	9	7.15	5 39 7.29334	.03371	-2 16 58.4454	.0926	-.039939	0.011583	-0.23589	0.02935
2146	51880	30	6.87	5 39 9.83316	.01506	-2 55 13.4645	.0698	0.374962	0.010345	1.42847	0.03036
2151	51925	31	7.65	5 39 22.34469	.00869	-2 49 19.8110	.0294	-.006974	0.004147	-0.96825	0.01297
2153	0	36	9.97	5 39 55.66049	.00726	-2 56 17.3586	.0263	-.001231	0.003625	-0.07678	0.01449
2155	52155	15	7.85	5 40 29.52448	.02787	-2 20 5.3557	.0992	-.020773	0.017748	0.08271	0.05515
2156	52194	46	9.05	5 40 42.16905	.00883	-2 32 31.1053	.0236	0.099431	0.006443	-2.81541	0.01727
2157	0	5	9.85	5 40 54.73335	.05137	-2 10 29.7342	.1576	-.033514	0.010450	1.12009	0.04339
2158	0	17	10.65	5 41 12.23093	.02207	-2 37 49.2130	.0761	-.048321	0.014447	-1.06801	0.06296
2159	0	17	10.30	5 41 24.43339	.01517	-2 42 24.7370	.0655	-.159410	0.012918	-1.69709	0.06461
2160	52375	50	8.65	5 41 33.58462	.00837	-2 30 15.4665	.0303	-.030532	0.008237	-0.58556	0.02650
2161	52385	13	8.25	5 41 35.44855	.02028	-2 5 45.6825	.0564	-.076113	0.016394	1.07748	0.03630
2162	52395	45	8.15	5 41 37.99922	.00660	-2 31 24.0433	.0324	-.095486	0.005998	-2.64175	0.03102
2163	52418	54	8.05	5 41 45.92202	.01523	-2 21 50.3717	.0538	-.017577	0.014783	-0.00156	0.04219
2164	52452	71	8.80	5 41 57.82682	.00786	-2 55 55.1445	.0258	-.031516	0.006464	-2.11837	0.02579
2165	0	18	10.30	5 42 3.48372	.01322	-2 31 2.2340	.0460	-.028639	0.007800	-1.71128	0.03083
2167	0	3	10.60	5 42 19.56335	.06349	-2 17 57.0597	.1534	0.010995	0.012074	-0.62281	0.03988
2168	52562	47	7.53	5 42 30.50607	.01185	-2 31 16.9011	.0391	-.029108	0.010716	-0.50225	0.03068
2169	0	22	10.13	5 42 43.36747	.01382	-2 20 27.4186	.0457	-.072983	0.007334	-0.94839	0.02892
2170	52618	47	8.27	5 42 47.94750	.00902	-2 46 7.3006	.0336	-.083924	0.007959	-0.25343	0.02948
2171	0	22	10.67	5 42 48.86780	.02317	-2 19 38.7226	.0644	-.097440	0.009624	0.35299	0.02351
2175	0	29	9.85	5 43 36.60554	.01147	-2 23 26.9065	.0719	-.076174	0.007603	-0.35476	0.05504
2176	0	19	10.10	5 43 38.65111	.01394	-2 29 6.0957	.0463	0.041190	0.004376	-0.65464	0.01941
2177	0	17	10.70	5 43 38.79486	.01503	-2 45 48.1473	.0587	0.068544	0.011547	-1.62543	0.05330
2178	0	18	10.75	5 43 40.16323	.01394	-2 30 34.2647	.0641	0.043955	0.006377	0.00525	0.05356
2179	0	3	10.40	5 43 45.86804	.02448	-2 13 56.7007	.1182	-.068749	0.015538	-2.67641	0.02254
2180	0	4	10.10	5 43 59.50431	.04811	-2 3 8.5450	.1238	-.151080	0.010248	-0.01078	0.03626
2182	0	4	10.40	5 44 17.01023	.04865	-2 3 35.5943	.0960	-.204273	0.009481	1.74444	0.02186
2229	0	7	9.90	5 18 46.30752	.01890	-3 33 9.4077	.0723	-.123806	0.006361	1.49481	0.01950
2231	0	7	10.20	5 19 17.40186	.02980	-3 33 43.4083	.0656	0.082529	0.009719	0.15986	0.01328
2232	0	3	11.50	5 19 19.26447	.03293	-3 35 39.1982	.1570	0.245350	0.006581	0.60844	0.03258
2233	0	10	10.10	5 19 22.01375	.04234	-3 8 14.9688	.0975	-.058037	0.011042	-1.31656	0.01773
2234	0	20	10.10	5 19 33.55285	.01539	-3 17 41.6618	.0497	0.000306	0.010239	-0.09013	0.02299

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
2235	0	7	9.90	5 19 35.54211	.02713	-3 33 15.1388	.1141	0.021234	0.008794	-0.40461	0.03530
2238	0	7	10.20	5 19 41.71979	.04851	-3 30 36.9982	.1494	0.031886	0.007765	1.18213	0.03005
2241	0	9	10.70	5 19 49.25697	.00815	-3 25 57.3138	.0671	-.160965	0.004087	1.66448	0.02451
2244	0	13	9.40	5 20 26.79260	.01643	-3 29 50.4956	.1012	-.003464	0.005039	-2.59263	0.03504
2245	0	25	10.70	5 20 39.87863	.01021	-3 12 42.2120	.0371	-.023376	0.004702	0.22045	0.01951
2246	48302	60	8.80	5 20 49.63028	.00559	-3 8 51.6462	.0214	-.085437	0.004358	0.48572	0.01482
2251	0	25	10.90	5 21 5.72133	.01057	-3 7 58.0031	.0313	-.004503	0.006435	0.38437	0.01685
2254	0	25	11.20	5 21 7.99810	.01206	-3 12 38.3432	.0372	0.039408	0.008454	-0.08186	0.02376
2256	0	19	9.70	5 21 15.37151	.00825	-3 22 33.6972	.0362	0.088832	0.004590	-0.68332	0.02127
2257	0	25	10.50	5 21 20.91528	.00815	-3 12 33.8173	.0373	-.008699	0.005693	-0.37910	0.01985
2258	0	25	10.50	5 21 21.69033	.01040	-3 7 4.3174	.0291	-.017424	0.006211	0.91753	0.01690
2259	48411	62	8.70	5 21 23.11753	.00581	-3 7 41.6292	.0219	-.066245	0.004623	0.28363	0.01614
2260	0	7	10.70	5 21 23.83412	.01965	-3 27 28.6382	.0855	0.035779	0.004731	-1.25449	0.02899
2263	0	7	11.00	5 21 29.90698	.01855	-3 28 6.3292	.0767	-.027268	0.002753	0.92552	0.02072
2264	0	7	10.60	5 21 33.13268	.02505	-3 34 12.3141	.1025	0.047104	0.007537	-1.18874	0.03428
2266	0	45	9.70	5 21 55.26590	.00737	-3 16 57.0669	.0253	-.036054	0.002497	0.39997	0.01355
2267	0	25	10.40	5 21 57.91138	.00871	-3 12 6.0542	.0258	-.000411	0.004933	-0.46872	0.00812
2268	48502	38	8.50	5 21 58.70269	.00507	-3 19 22.9107	.0227	0.050483	0.003582	-0.31789	0.01114
2270	0	13	9.00	5 22 3.35089	.01395	-3 33 8.8287	.0931	-.040190	0.004507	-0.54661	0.03367
2271	0	33	10.40	5 22 21.22707	.00675	-3 24 4.7645	.0231	0.046886	0.004281	-0.02398	0.01379
2272	0	42	10.70	5 22 23.04884	.00872	-3 3 22.6236	.0254	-.026772	0.004934	-0.49847	0.01616
2273	0	10	11.20	5 22 36.87818	.02264	-3 38 45.5582	.0914	-.039982	0.019875	0.75613	0.06218
2274	0	12	10.20	5 22 36.67523	.02203	-3 52 44.5273	.0625	-.021813	0.016920	0.20906	0.05808
2275	0	19	10.30	5 22 51.47336	.00818	-3 29 54.2949	.0285	0.018079	0.004216	0.40252	0.01546
2276	0	12	11.20	5 22 52.59705	.01331	-3 29 22.7231	.0396	-.026480	0.005479	1.40626	0.01551
2277	0	12	10.00	5 22 54.20965	.02356	-3 51 38.4026	.0652	0.058669	0.011555	-4.25856	0.05543
2278	0	36	10.50	5 22 55.40850	.01076	-3 4 4.5416	.0327	0.054650	0.009811	-0.68486	0.01656
2279	0	35	10.80	5 22 56.69031	.00691	-3 20 13.7613	.0236	-.030765	0.003013	-0.98800	0.01213
2280	0	12	10.40	5 22 57.35879	.01796	-3 37 31.5052	.0799	-.000529	0.011495	0.63805	0.08121
2281	0	19	10.50	5 23 2.88496	.00870	-3 32 47.4304	.0284	0.007323	0.004911	1.54989	0.01557
2282	48691	32	8.80	5 23 4.45068	.01428	-3 55 34.6757	.0540	-.025074	0.012440	-3.14412	0.04873
2283	0	13	10.80	5 23 10.72523	.02131	-3 40 1.0267	.0592	-.054440	0.019730	0.31376	0.05830
2284	0	19	10.40	5 23 11.92662	.00819	-3 27 5.2990	.0274	0.014434	0.004146	0.44142	0.01607

Table 9: (Continued)

AC#	ACRS#	N	m	α	$\epsilon\alpha$	δ	ϵ_δ	$\mu\alpha$	$\epsilon\mu\alpha$	μ_δ	$\epsilon\mu_\delta$
2286	0	66	9.30	5 23 19.96924	.00660	-3 3 16.2686	.0197	0.027380	0.003763	0.27673	0.01292
2289	0	27	10.50	5 23 40.52388	.00583	-3 22 45.1697	.0318	-.000186	0.000946	0.83443	0.01369
2290	48830	64	8.40	5 23 46.13670	.00495	-3 7 6.6309	.0174	0.039003	0.004038	-0.48238	0.01193
2291	0	45	9.50	5 24 2.64368	.00587	-3 30 49.3630	.0261	-.006733	0.003997	1.53215	0.02318
2292	0	21	9.90	5 24 7.19945	.01303	-3 46 52.9625	.0291	-.047293	0.009120	0.41050	0.00458
2293	0	12	11.10	5 24 12.89207	.01250	-3 27 1.1146	.0363	-.018233	0.004278	4.72574	0.01489
2294	0	33	9.60	5 24 13.15395	.00712	-3 37 25.1654	.0408	-.038516	0.001790	-1.23108	0.03397
2295	0	16	11.50	5 24 14.98828	.01309	-3 28 51.2684	.0297	0.022762	0.008994	0.27488	0.00966
2296	0	19	11.20	5 24 15.33853	.00848	-3 34 41.2213	.0288	0.051243	0.004809	0.95787	0.01558
2297	0	24	10.80	5 24 17.02185	.01017	-3 7 19.5227	.0397	0.009582	0.006091	-0.47824	0.02341
2298	0	19	11.30	5 24 19.56665	.00816	-3 27 46.3306	.0314	0.050278	0.003688	-2.10067	0.01529
2299	0	27	9.80	5 24 28.60361	.00594	-3 35 52.0339	.0298	0.029921	0.002968	1.51026	0.01707
2300	0	25	10.40	5 24 43.26704	.00872	-3 9 50.0098	.0324	-.025835	0.003694	0.52303	0.01858
2301	0	13	11.50	5 24 43.23153	.01857	-3 38 42.5864	.0773	-.060495	0.011185	-1.51913	0.06759
2302	0	12	11.30	5 24 48.51661	.01413	-3 51 59.6571	.0883	-.048513	0.004674	-1.12938	0.06414
2303	0	13	10.20	5 24 49.47897	.02167	-3 43 15.7546	.0724	-.004291	0.016275	-0.85039	0.03196
2304	0	6	10.60	5 24 57.55076	.01740	-3 40 8.2297	.0979	0.013609	0.013727	-5.34266	0.07392
2305	0	13	10.50	5 24 58.85859	.01539	-3 37 23.5781	.0772	-.001242	0.011636	-1.38938	0.06059
2306	0	37	10.10	5 25 19.81550	.00896	-3 8 7.6623	.0367	0.141250	0.004399	2.20280	0.01655
2307	0	37	10.50	5 25 22.04821	.01259	-3 7 4.3208	.0772	0.093466	0.005940	-1.03585	0.02796
2308	0	81	9.30	5 25 22.63698	.00447	-3 7 3.8600	.0178	0.026233	0.002119	0.51593	0.01089
2309	0	48	9.30	5 25 24.29668	.00880	-3 38 14.4200	.0599	-.090488	0.001992	-1.19629	0.05639
2310	0	47	11.10	5 25 37.56065	.00800	-3 15 39.2805	.0280	0.045410	0.003970	0.91413	0.02143
2311	0	17	11.30	5 25 37.93648	.01380	-3 50 12.9498	.0649	-.184532	0.003894	-2.87946	0.05695
2312	0	23	11.00	5 25 51.71203	.02664	-3 49 15.8607	.0810	-.101156	0.017271	-0.99480	0.04891
2313	0	61	9.90	5 26 0.18756	.00870	-3 3 38.8782	.0219	0.036906	0.004974	0.42098	0.01487
2314	49298	64	9.40	5 26 8.62368	.00504	-3 16 29.9628	.0154	0.075484	0.002291	-0.27870	0.00816
2315	49304	16	8.50	5 26 11.71289	.01738	-3 54 57.0697	.0571	0.159493	0.013047	-8.19140	0.05126
2316	0	78	9.20	5 26 12.56556	.00567	-3 5 15.8499	.0178	0.059778	0.003992	-0.56871	0.01237
2317	0	8	11.50	5 26 16.58535	.02896	-3 37 48.9727	.0647	-.127292	0.013977	2.03555	0.03731
2318	0	12	11.50	5 26 18.95305	.02451	-3 42 31.7580	.0646	-.094283	0.013432	-0.79500	0.03559
2319	0	15	11.50	5 26 20.10300	.01501	-3 16 5.0675	.0450	0.265218	0.006172	1.99331	0.01890
2320	0	9	11.30	5 26 24.35973	.02614	-3 38 50.8459	.0653	-.348572	0.021122	-3.43237	0.03286

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
2321	49347	54	6.20	5 26 27.04149	.00645	-3 20 47.2764	.0325	-.128508	0.003030	-1.79962	0.01531
2322	0	29	11.50	5 26 31.17343	.01032	-3 10 56.5539	.0309	0.056405	0.005827	0.23022	0.01736
2323	0	13	11.30	5 26 43.17683	.01799	-3 40 57.5087	.0670	-.076790	0.012675	-0.51376	0.03986
2324	49426	36	7.10	5 26 54.02143	.01228	-3 29 5.5630	.0814	-.250410	0.004754	-1.19912	0.03137
2325	0	33	11.50	5 26 58.65976	.00777	-3 20 26.4244	.0239	0.026917	0.003769	0.15602	0.01097
2326	0	13	11.50	5 27 1.25330	.01912	-3 40 41.2609	.0600	-.078802	0.012558	-0.28477	0.03259
2327	0	11	11.00	5 27 3.77797	.02044	-3 49 16.2041	.0609	-.078868	0.014202	0.99858	0.03417
2328	0	27	10.60	5 27 10.86681	.01065	-3 7 24.5614	.0285	-.000963	0.007267	-0.22024	0.01877
2329	0	11	10.50	5 27 16.91920	.01559	-3 48 16.7698	.0483	-.075996	0.003225	-0.38833	0.02882
2331	0	24	11.50	5 27 30.06965	.01196	-3 23 54.4608	.0294	-.174548	0.005703	-4.33449	0.01998
2332	0	18	11.50	5 27 41.05656	.01082	-3 4 9.1490	.0386	0.039552	0.004948	-1.40142	0.02012
2333	0	18	11.60	5 27 43.31785	.01814	-3 4 15.4302	.0563	-.045121	0.008577	-0.25872	0.04277
2334	49610	96	9.20	5 27 51.99793	.00365	-3 20 12.8913	.0152	-.096789	0.002025	1.25612	0.00790
2335	0	20	11.00	5 27 53.21318	.01978	-3 47 59.6275	.0371	-.055510	0.013821	-0.24671	0.01102
2336	0	60	9.10	5 27 56.50989	.01058	-3 38 53.6111	.0320	-.031221	0.009365	-1.95467	0.02366
2337	0	25	11.00	5 28 0.39086	.01884	-3 46 39.4322	.0577	-.086440	0.015299	-0.57628	0.05500
2338	0	35	11.40	5 28 8.81568	.00714	-3 34 5.6965	.0251	-.040422	0.003667	-0.08104	0.01388
2339	0	42	11.50	5 28 14.36500	.01015	-3 23 37.1527	.0349	-.248371	0.009004	1.32907	0.03161
2340	0	37	10.90	5 28 15.81376	.00584	-3 31 36.0754	.0213	-.088331	0.003354	0.93661	0.01308
2341	0	33	10.20	5 28 16.99076	.00693	-3 19 24.7278	.0234	-.109598	0.003316	-0.64202	0.01295
2342	0	37	11.50	5 28 24.55607	.01280	-3 20 53.6137	.0568	-.114731	0.005577	2.96517	0.02334
2343	0	32	10.20	5 28 26.41317	.01871	-3 13 42.7011	.0763	-.175580	0.007603	2.39620	0.03269
2344	0	23	11.20	5 28 27.39018	.01583	-3 49 32.6374	.0642	-.130514	0.013586	0.36770	0.05776
2345	0	38	11.50	5 28 29.18237	.01084	-3 9 53.9934	.0320	-.019920	0.006439	0.30717	0.01837
2346	0	29	10.40	5 28 29.25238	.00973	-3 37 27.1374	.0390	-.057615	0.003355	0.15199	0.02944
2347	49742	143	8.80	5 28 33.81821	.00269	-3 21 29.5921	.0103	-.544995	0.001785	-11.56757	0.00745
2348	0	7	10.40	5 28 35.54662	.01922	-3 58 11.9968	.0792	-.036540	0.007274	0.30388	0.03173
2349	49757	107	8.80	5 28 38.20078	.00438	-3 14 2.1806	.0134	-.125934	0.003302	0.31218	0.00930
2350	0	34	11.20	5 28 38.49422	.00872	-3 36 28.0064	.0219	-.048669	0.005603	0.41961	0.01398
2351	0	25	11.00	5 28 45.45092	.01365	-3 39 41.7612	.0482	-.149971	0.009593	-4.13035	0.04337
2352	0	49	11.50	5 28 46.89076	.00979	-3 20 9.2734	.0285	-.131299	0.004065	-0.29211	0.01873
2354	0	35	10.70	5 28 56.02096	.00895	-3 34 5.0682	.0219	-.077277	0.004185	-0.85550	0.00910
2355	0	37	10.80	5 29 5.27487	.00720	-3 31 42.1412	.0258	-.079858	0.002995	-0.50410	0.01365

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
2356	0	22	11.00	5 29 7.77435	.01664	-3 52 10.5095	.0725	-.008763	0.006403	-0.60531	0.05313
2357	0	8	10.10	5 29 8.58495	.03863	-3 29 11.6665	.1023	-.104817	0.013464	9.64611	0.04260
2358	0	39	11.20	5 29 8.95344	.01358	-3 15 43.5882	.0364	-.095539	0.005460	1.12602	0.02682
2359	49871	79	7.60	5 29 13.67131	.00568	-3 15 7.8176	.0210	-.004492	0.004472	-0.82217	0.01726
2360	0	29	10.40	5 29 19.47870	.00933	-3 3 50.0533	.0330	0.109439	0.005670	0.47208	0.01808
2361	0	13	10.40	5 29 23.19807	.02061	-3 39 16.1287	.0795	-.052681	0.016541	-1.89868	0.06400
2362	0	31	11.20	5 29 25.42304	.00720	-3 23 57.1412	.0247	-.057493	0.003530	0.20936	0.01138
2363	0	13	10.70	5 29 30.10971	.01896	-3 38 27.8959	.0497	-.061345	0.014994	-0.28746	0.02627
2365	0	65	9.50	5 29 32.95368	.00540	-3 11 59.1841	.0178	0.061784	0.003338	-1.89193	0.01142
2366	0	29	11.10	5 29 34.33532	.00993	-3 7 35.0125	.0419	0.018862	0.004537	-3.32416	0.03248
2367	0	30	11.20	5 29 41.36075	.01349	-3 23 52.1775	.0304	-.089746	0.006399	0.36073	0.01336
2368	0	26	11.10	5 29 42.18900	.01033	-3 7 23.1107	.0400	-.031327	0.006715	0.16085	0.02599
2369	0	25	10.40	5 29 45.04334	.00779	-3 17 53.3102	.0301	-.003943	0.003399	-0.24137	0.01217
2370	0	63	9.00	5 29 47.05704	.00565	-3 35 57.0234	.0186	-.117888	0.003111	0.59317	0.00987
2371	0	12	10.70	5 29 47.67842	.01745	-3 53 11.3786	.0709	0.145598	0.011209	-5.03807	0.05981
2372	0	25	11.20	5 29 48.79756	.00775	-3 32 57.9469	.0271	-.058695	0.003340	-1.66022	0.01015
2373	0	25	11.20	5 29 49.82620	.00782	-3 18 56.6782	.0282	-.049338	0.003434	-0.84322	0.01302
2374	0	20	11.50	5 29 55.29516	.01133	-3 0 15.3239	.0456	-.058718	0.004974	-0.35819	0.02471
2375	0	22	11.40	5 29 59.24750	.01049	-2 59 40.6855	.0377	0.112471	0.005699	-2.27157	0.02163
2376	0	29	10.40	5 30 6.69059	.01024	-3 4 24.5126	.0367	0.042299	0.007482	-2.20978	0.02687
2377	0	13	11.10	5 30 7.66106	.01891	-3 49 21.7995	.0488	-.095756	0.010026	0.46056	0.03596
2378	0	30	10.10	5 30 12.03090	.01130	-3 14 5.8090	.0308	0.000756	0.008102	-0.65009	0.01924
2379	0	26	10.80	5 30 12.53325	.00882	-3 19 37.6535	.0291	0.026563	0.003612	-2.06775	0.01167
2380	0	34	10.70	5 30 22.67622	.00621	-3 21 40.4213	.0247	0.012854	0.003080	0.29459	0.00908
2381	0	72	8.80	5 30 23.32054	.00578	-3 7 10.5153	.0197	0.044248	0.003540	-0.82167	0.01110
2382	0	26	10.70	5 30 25.65545	.00772	-3 16 38.7673	.0204	0.167283	0.003395	-0.57154	0.01013
2383	0	26	10.50	5 30 30.34818	.00706	-3 16 25.4003	.0282	0.004169	0.003233	-0.42946	0.01222
2384	0	25	11.50	5 30 32.19733	.00938	-2 51 17.9988	.0266	-.002840	0.006906	-1.34303	0.01564
2385	0	14	11.00	5 30 31.84396	.02354	-3 40 38.7481	.0587	-.136027	0.021697	0.19788	0.04447
2386	0	26	10.70	5 30 37.13198	.00668	-3 16 58.3736	.0217	0.010710	0.003161	-0.15524	0.01174
2387	0	29	11.30	5 30 46.82818	.00928	-3 3 26.1412	.0307	0.104086	0.005105	-1.26436	0.01996
2388	0	20	11.30	5 30 38.80138	.01188	-3 27 35.0210	.0375	-.069712	0.005792	-0.11568	0.01535
2389	0	23	11.20	5 30 38.90026	.01199	-3 32 51.9458	.0427	-.094143	0.003202	0.66366	0.01609

Table 9: (Continued)

AC#	ACRS#	N	m	α			ϵ_α	δ			ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
2390	0	33	11.20	5	30	51.30787	.00635	-3	23	25.0850	.0208	-.007048	0.003402	0.11745	0.00952
2391	0	25	11.20	5	30	52.61563	.01798	-3	33	4.9071	.0232	-.074135	0.006906	-1.55322	0.00672
2392	0	33	10.70	5	30	54.34493	.00477	-3	20	26.0992	.0216	-.026497	0.001926	-0.25355	0.01054
2393	50197	89	8.60	5	31	0.53048	.00445	-3	8	34.9865	.0124	0.076216	0.002648	-2.09919	0.00739
2394	50214	59	8.40	5	31	8.00518	.00603	-3	29	47.6924	.0273	-.036201	0.004037	-0.59142	0.02116
2395	0	46	9.30	5	31	11.38183	.01761	-3	39	14.4108	.0317	-.057664	0.017462	-1.38161	0.02953
2396	0	33	11.00	5	31	12.56878	.00747	-3	34	47.1427	.0224	0.007612	0.003453	-1.81344	0.01207
2397	0	31	11.40	5	31	20.87085	.00958	-2	58	20.8401	.0411	0.096520	0.005222	-1.63988	0.01970
2398	0	35	10.80	5	31	21.85566	.00574	-3	25	52.0088	.0218	-.065606	0.001552	-0.05776	0.01125
2399	0	39	10.70	5	31	25.04438	.00726	-3	11	27.1190	.0294	0.012364	0.004005	0.16018	0.01628
2400	0	69	9.60	5	31	30.64484	.00543	-3	1	25.5758	.0221	0.024839	0.002566	-0.36141	0.01081
2401	0	6	11.20	5	31	30.73775	.06011	-3	39	50.5216	.2021	0.163394	0.017009	-5.13408	0.09571
2402	0	33	10.50	5	31	31.90697	.00809	-3	31	47.5295	.0232	0.035338	0.003855	-1.52010	0.01305
2403	0	30	11.50	5	31	36.06461	.00986	-3	9	2.4071	.0350	0.051785	0.006090	-0.29807	0.01540
2404	0	31	11.40	5	31	38.60757	.00889	-3	13	21.0569	.0383	0.070327	0.006453	0.28235	0.02584
2405	0	66	9.60	5	31	40.93130	.00465	-3	16	40.9701	.0147	0.009486	0.002439	-0.28201	0.00872
2406	0	41	8.90	5	31	40.59769	.00896	-3	28	35.2227	.0309	-.171055	0.003311	-3.52037	0.01291
2407	0	37	11.40	5	31	53.28898	.00709	-3	5	48.4093	.0324	0.070326	0.005061	0.99799	0.01925
2408	0	9	11.10	5	31	53.18763	.01331	-3	48	0.1974	.0597	-.107299	0.008015	0.58237	0.05686
2409	0	21	10.80	5	31	53.69207	.00932	-3	28	44.8243	.0274	-.034207	0.004257	2.58020	0.00892
2410	0	43	11.50	5	31	55.06900	.00956	-2	57	34.0163	.0296	0.084643	0.006892	-1.12953	0.01420
2411	0	36	11.50	5	32	4.05268	.01085	-2	58	24.2244	.0428	0.174270	0.007855	-0.10778	0.02732
2412	0	31	11.40	5	32	4.09760	.01144	-3	6	39.7308	.0458	0.087510	0.007654	-0.90054	0.02091
2413	50417	25	8.90	5	32	11.36454	.01978	-3	42	41.2460	.0682	0.125539	0.018187	-0.79816	0.06921
2414	0	9	11.20	5	32	12.49665	.02635	-3	43	52.3076	.0743	-.072071	0.019241	0.43106	0.05938
2415	0	23	11.50	5	32	26.09144	.01413	-3	18	17.7310	.0508	0.057900	0.006630	0.48090	0.02409
2416	0	9	11.30	5	32	33.08315	.03077	-3	53	46.7667	.0518	-.190435	0.017952	0.62697	0.02801
2417	0	17	10.60	5	32	36.08900	.00913	-3	34	36.9696	.0268	-.028395	0.004487	-1.08385	0.01457
2418	50514	25	9.00	5	32	39.72112	.01604	-3	44	30.4239	.1058	-.058043	0.015693	-3.44730	0.11177
2419	0	42	9.50	5	32	43.85204	.00881	-3	13	55.4597	.0262	0.078378	0.006291	-0.28350	0.01593
2420	0	18	11.50	5	32	51.58177	.01307	-3	0	31.9041	.0413	0.046015	0.008509	1.32568	0.03426
2421	0	17	10.10	5	32	59.25626	.01082	-3	25	53.7625	.0305	-.086588	0.006971	-0.36893	0.01524
2422	0	18	11.50	5	32	59.97395	.02014	-3	2	22.3574	.1139	0.163868	0.008301	0.21077	0.04548

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	ϵ_{μ_α}	μ_δ	ϵ_{μ_δ}
2423	0	18	11.60	5 33 0.69734	.01299	-3 22 9.5545	.0414	-.126793	0.011368	0.19166	0.02851
2424	0	14	11.50	5 33 1.48109	.01050	-3 24 40.0136	.0533	0.040912	0.004868	1.00465	0.02378
2426	50629	34	7.30	5 33 5.92875	.01353	-3 17 1.9988	.0449	-.054594	0.009030	-0.42319	0.02525
2427	0	16	11.50	5 33 8.17773	.01523	-3 4 27.4415	.0471	0.184230	0.010899	1.67932	0.04375
2428	0	26	8.80	5 33 7.85991	.00786	-3 36 33.8979	.0373	-.051946	0.006405	-0.13743	0.03351
2429	50649	71	8.90	5 33 9.96334	.00468	-3 20 49.4932	.0184	0.086060	0.003549	0.93263	0.01172
2430	0	60	9.90	5 33 14.51801	.00623	-3 5 23.7131	.0311	0.134995	0.004440	2.05394	0.01258
2431	0	21	11.50	5 33 17.30051	.00934	-3 10 16.9302	.0401	0.078288	0.006935	0.93602	0.02596
2432	0	16	11.50	5 33 18.61450	.01371	-3 0 34.6647	.0499	0.179756	0.008845	3.05033	0.03461
2433	0	9	10.00	5 33 20.81761	.01534	-3 39 42.4721	.0686	-.073239	0.008755	0.67342	0.05122
2434	0	9	10.60	5 33 23.06787	.01954	-3 36 11.0538	.0861	0.120846	0.017609	0.89762	0.07837
2435	0	11	10.90	5 33 25.34624	.01485	-3 35 39.3264	.0434	-.140910	0.008236	0.11391	0.01132
2436	0	21	11.50	5 33 26.42177	.01094	-3 12 42.7525	.0398	0.212437	0.009149	0.57164	0.02476
2437	0	26	10.60	5 33 33.88684	.00852	-3 22 43.1859	.0303	0.145335	0.005278	-0.12532	0.01230
2438	0	11	11.60	5 33 34.21047	.01151	-3 27 11.7723	.0468	0.031018	0.006636	-1.64952	0.03058
2439	0	18	11.50	5 33 36.05038	.01424	-3 23 40.1078	.0448	0.045870	0.007949	-0.82493	0.03191
2440	0	29	11.50	5 33 43.61579	.01086	-2 56 9.5194	.0310	-.029562	0.007364	-0.62619	0.01499
2441	0	18	11.50	5 33 43.81387	.01011	-3 12 4.0591	.0275	0.017031	0.006739	1.41983	0.02366
2443	0	21	11.40	5 33 45.62572	.01478	-3 19 2.3186	.0270	-.043805	0.012531	-0.62320	0.01627
2444	0	18	11.50	5 33 46.17047	.01017	-3 10 49.7513	.0513	0.245541	0.006952	-0.28498	0.03088
2445	0	15	11.50	5 33 48.25205	.01260	-3 16 41.2168	.0251	0.170762	0.007925	-0.69379	0.01308
2446	0	20	11.50	5 33 52.33208	.01222	-3 3 57.7131	.0513	0.199600	0.010020	0.01101	0.04832
2447	0	15	11.50	5 33 58.13456	.02705	-3 26 9.4089	.0488	-.089108	0.010031	-0.37491	0.03334
2448	0	11	11.50	5 33 58.34281	.01353	-3 27 57.4697	.0428	0.080530	0.008651	-0.50034	0.02794
2449	0	9	11.50	5 33 59.07708	.02510	-3 36 46.7718	.0899	-.188935	0.014262	3.24014	0.07477
2450	0	18	11.50	5 34 1.06652	.00946	-3 3 9.5215	.0529	-.038279	0.006142	-0.33396	0.04547
2451	0	16	9.60	5 34 3.99532	.01106	-3 44 1.8481	.0695	-.069482	0.006259	0.86992	0.06726
2452	0	17	10.40	5 34 6.45453	.00891	-3 35 50.4184	.0322	0.253786	0.004614	-1.99338	0.01672
2453	0	26	11.00	5 34 7.27740	.00849	-3 18 42.3356	.0236	0.095154	0.004650	0.36519	0.01162
2454	0	18	10.20	5 34 7.92347	.01271	-3 13 53.7966	.0391	0.061769	0.005239	1.11192	0.01940
2455	50868	76	9.60	5 34 12.06359	.00467	-3 19 26.1468	.0179	0.271424	0.003763	0.70969	0.01159
2456	0	38	9.20	5 34 12.10450	.00777	-3 31 59.3142	.0344	0.045527	0.004501	0.17838	0.01659
2457	0	19	10.90	5 34 13.83152	.00844	-3 13 55.0114	.0444	0.189823	0.004618	1.03769	0.02029

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
2458	0	16	10.50	5 34 14.70700	.01095	-3 30 26.2391	.0489	-.137288	0.006170	-0.09145	0.02708
2459	0	15	11.50	5 34 15.10593	.01029	-3 13 42.6050	.0348	0.095626	0.004977	1.65970	0.01855
2460	0	18	11.50	5 34 15.22121	.01466	-3 11 35.2069	.0412	0.245685	0.006666	-2.83445	0.01254
2461	0	19	10.60	5 34 17.51861	.01195	-3 7 27.2126	.0668	0.167623	0.005494	2.64210	0.02620
2462	0	23	11.50	5 34 18.04139	.00755	-3 23 32.0354	.0299	0.142640	0.006383	0.26857	0.02067
2463	0	23	11.50	5 34 19.93600	.00815	-3 21 1.7559	.0340	0.051108	0.003570	0.08109	0.02162
2464	0	20	11.20	5 34 21.43458	.01621	-3 10 45.5230	.0570	0.008886	0.006820	0.43749	0.02492
2465	0	33	10.50	5 34 27.67445	.00827	-3 2 32.1256	.0309	0.248464	0.004439	0.97921	0.01969
2466	0	9	10.40	5 34 27.07692	.02916	-3 47 16.3215	.0758	-.030505	0.022936	-0.96915	0.05086
2467	0	26	10.90	5 34 29.38385	.00704	-3 23 48.8115	.0288	0.101880	0.004071	-0.03590	0.01389
2469	0	10	10.70	5 34 36.01596	.02007	-3 54 19.7198	.2042	-.389487	0.008858	3.49866	0.05587
2470	0	26	11.40	5 34 40.77068	.00963	-3 12 15.2555	.0565	0.153405	0.005744	0.58349	0.01763
2471	0	22	11.20	5 34 43.20762	.01048	-3 27 41.0998	.0427	0.054453	0.005392	0.96910	0.02202
2472	0	22	10.40	5 34 44.55432	.01044	-3 32 53.5990	.0276	0.030961	0.003946	1.39612	0.01466
2473	0	25	9.30	5 34 44.43140	.01509	-3 52 18.8131	.0742	-.177629	0.010823	1.07148	0.05328
2474	0	13	11.50	5 34 47.45973	.01564	-3 36 26.0758	.0759	-.171159	0.007092	-0.60952	0.04888
2478	0	7	9.20	5 35 8.61335	.03282	-3 41 0.1280	.1277	-.143346	0.020205	1.21665	0.02941
2480	0	5	10.90	5 35 18.97697	.01789	-3 36 13.5921	.0977	-.054529	0.007773	0.83712	0.02859
2481	0	3	11.40	5 35 20.13706	.05342	-3 39 15.1489	.1618	-.182237	0.023625	1.41401	0.05930
2483	0	3	11.60	5 35 38.00399	.02698	-3 44 18.5249	.3467	-.038625	0.004239	0.82227	0.07443
2484	0	26	11.30	5 35 41.39703	.00941	-3 16 59.2212	.0299	0.181925	0.005876	0.76273	0.01286
2485	0	34	9.90	5 35 43.95311	.01771	-3 23 2.1355	.0427	0.080175	0.006174	-1.35073	0.01450
2486	0	66	9.70	5 35 44.43681	.00518	-3 21 28.5694	.0154	0.247991	0.002863	-0.82826	0.00844
2488	51210	58	9.30	5 35 49.24120	.00737	-3 34 57.9304	.0305	0.142515	0.003277	-0.42031	0.01245
2489	0	30	11.40	5 35 55.95269	.01021	-3 15 52.7666	.0344	0.263899	0.006426	0.50070	0.02436
2490	0	32	10.40	5 35 58.33882	.00689	-3 5 13.4933	.0241	0.280232	0.003773	1.41420	0.01454
2491	0	3	11.20	5 36 10.70489	.05406	-3 38 58.9683	.2515	-.052674	0.019443	0.50088	0.08766
2492	0	3	11.50	5 36 22.90298	.04043	-3 37 35.4957	.2397	-.101905	0.009828	-0.84432	0.05874
2494	0	3	11.00	5 36 31.76315	.14709	-3 49 55.2587	.8064	0.001401	0.024799	-0.26288	0.14795
2495	0	32	10.40	5 36 46.70209	.01070	-3 9 13.6740	.0279	0.185709	0.005957	0.90543	0.01565
2497	0	3	11.00	5 36 57.75064	.07843	-3 48 38.7759	.4649	-.035976	0.012706	0.95063	0.07852
2498	51444	31	6.30	5 37 1.54602	.01320	-3 35 27.5331	.0893	-.063528	0.005116	-0.57021	0.03628
2499	0	15	11.60	5 37 3.87498	.01203	-3 33 4.8934	.0522	-.022077	0.004632	0.06755	0.02245

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
2500	0	3	11.30	5 37 12.02722	.07447	-3 37 5.1234	.2512	-.232655	0.028577	-0.28344	0.07131
2501	0	15	10.50	5 37 12.38070	.02260	-3 35 33.1145	.0860	0.138519	0.008405	0.16948	0.03034
2502	0	13	10.70	5 37 14.09232	.01574	-3 25 36.0115	.0826	-.026995	0.005162	0.26951	0.02870
2503	0	7	11.20	5 37 17.27785	.03023	-3 0 32.4934	.1295	0.320048	0.006008	-2.20634	0.03687
2504	51517	79	7.40	5 37 25.54436	.00428	-3 21 22.9590	.0189	0.075446	0.002218	0.50584	0.00923
2506	0	49	10.00	5 37 41.93800	.00741	-3 7 0.4912	.0234	0.220850	0.004405	-0.06032	0.01140
2507	0	48	11.00	5 37 42.13378	.00673	-3 10 15.3043	.0249	0.006520	0.002404	0.34375	0.01386
2508	0	21	10.50	5 37 47.36999	.00746	-3 29 46.8089	.0363	-.072913	0.002021	-0.37115	0.01507
2509	51615	56	7.70	5 37 49.96798	.00804	-3 27 8.8792	.0311	-.080506	0.004170	0.31095	0.01923
2510	0	19	11.20	5 37 53.51823	.01570	-3 29 13.2057	.0492	-.016432	0.006487	0.90168	0.02618
2511	0	16	11.60	5 37 54.37659	.03100	-3 31 10.8367	.0701	-.256650	0.011236	-5.61900	0.02791
2512	0	9	11.60	5 37 55.49351	.02864	-3 37 50.9204	.1050	-.154330	0.015617	0.03459	0.03563
2515	51728	14	9.00	5 38 28.38582	.01782	-3 52 4.0118	.0775	-.017495	0.010615	-0.62221	0.07133
2516	51767	14	8.30	5 38 38.60847	.01516	-3 39 25.1369	.0499	-.036373	0.011763	-0.84361	0.04298
2517	0	4	10.90	5 38 43.15253	.03762	-3 36 42.4979	.1245	0.065669	0.013977	0.16219	0.04480
2518	0	25	11.20	5 38 54.16741	.00909	-3 26 12.9239	.0294	-.046463	0.004080	-0.62689	0.01455
2520	0	8	11.70	5 38 58.79517	.04368	-3 28 24.9432	.1229	0.226079	0.012902	-0.44825	0.04159
2521	0	15	10.70	5 39 2.09517	.01086	-3 30 8.8707	.0315	-.098050	0.003607	-0.10749	0.01119
2523	51861	13	7.50	5 39 6.60449	.01909	-3 45 17.7727	.0600	0.088348	0.010669	-1.31769	0.05135
2525	0	26	10.60	5 39 12.06509	.00998	-3 13 45.9160	.0312	0.014410	0.005943	-0.35563	0.01672
2526	0	15	11.00	5 39 13.85572	.01461	-3 32 10.1857	.0493	-.123086	0.004594	-1.79617	0.01784
2527	51914	43	9.50	5 39 19.92250	.00828	-3 30 57.4210	.0233	-.143160	0.003424	0.20541	0.01082
2528	0	3	11.40	5 39 29.78832	.02808	-3 44 46.8770	.1023	-.045684	0.011175	0.05980	0.06125
2529	0	12	11.00	5 39 40.58834	.01528	-3 20 52.7395	.0615	-.067270	0.003416	-0.81595	0.01690
2530	0	19	9.70	5 39 43.43073	.01996	-3 5 27.1977	.0721	0.213294	0.007139	2.42048	0.02505
2531	0	3	10.20	5 39 44.24197	.05636	-3 49 43.3543	.0221	0.009419	0.012893	-0.12003	0.01644
2532	0	13	10.00	5 39 48.46952	.01292	-3 15 44.8395	.0453	-.049914	0.002916	-0.99328	0.01684
2533	52017	32	8.80	5 39 50.56350	.01440	-3 20 40.6626	.0583	-.114966	0.011918	-0.48330	0.05415
2535	0	3	11.60	5 40 5.14995	.01900	-3 29 18.1971	.0890	-.097670	0.005272	-0.88117	0.02013
2536	0	11	10.40	5 40 8.02488	.02054	-3 25 39.2045	.0450	-.054645	0.013758	-0.64198	0.01037
2538	52119	42	9.50	5 40 22.05454	.00796	-3 13 24.8685	.0281	-.043546	0.005785	0.01519	0.02009
2541	0	9	9.90	5 40 51.99919	.02064	-2 59 54.4827	.0734	0.119702	0.006990	4.07891	0.02949
2543	0	27	9.90	5 41 1.58199	.01217	-3 12 58.9426	.0443	-.079034	0.002348	-0.67275	0.03439

Table 9: (Continued)

AC#	ACRS#	N	m	α	ϵ_α	δ	ϵ_δ	μ_α	$\epsilon\mu_\alpha$	μ_δ	$\epsilon\mu_\delta$
2554	0	7	10.20	5 42 35.65263	.03199	-3 0 33.7104	.0803	0.031572	0.013802	-2.76302	0.04641
2556	0	10	10.00	5 42 43.71042	.02364	-3 5 55.1489	.0503	-.000691	0.010220	1.35493	0.04836
2560	0	9	10.10	5 43 28.64793	.02239	-3 14 3.5456	.0426	-.126904	0.006628	0.13057	0.00626
2561	0	6	9.90	5 43 37.75728	.02659	-3 19 15.2818	.0795	-.183151	0.007677	0.72748	0.02273
2566	0	6	10.30	5 44 4.78693	.02995	-3 22 46.1517	.0718	-.169689	0.004641	0.09653	0.03225
2568	52924	28	9.10	5 44 16.61372	.00896	-3 16 39.6122	.0310	-.067543	0.005081	0.69636	0.02894
2570	0	25	8.50	5 44 17.84639	.00978	-3 16 56.1502	.0385	-.079858	0.005187	0.54531	0.03707
2574	52992	18	8.60	5 44 36.36885	.01943	-3 21 35.8774	.0538	-.059330	0.018029	-0.35152	0.04800
2576	0	4	10.40	5 44 41.50057	.09045	-3 18 13.5212	.1572	-.143084	0.015682	-0.79229	0.03043

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BIOGRAPHICAL SKETCH

Richard Laurence Smart was born in London, England, on the eighteenth of July 1963. He received his high school education from Alperton High School where he graduated in 1982 with two 'A' levels and five 'O' levels. He proceeded to Preston Polytechnic where in 1986 he received a joint honors upper second B.S. degree in mathematics and astronomy.

In 1987 he enrolled at the University of Florida, whereupon he became 'Ricky'. In 1990 he received his M.S. from the University of Florida and entered into the Ph.D. program. He is a member of the American Astronomical Society and a junior fellow of the Royal Astronomical Society.

He has always been very active in local organizations, enjoys a very socially aware political ideology and endeavor's to live up to those beliefs. His interests include flying, volleyball, skiing, reading, scuba diving and soccer.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Heinrich K. Eichhorn, Chair
Professor of Astronomy

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Haywood Smith
Associate Professor of Astronomy

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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December 1993

Dean, Graduate School